微分積分 II 配布資料 (演習問題) No.3　　　（作成 田嶋）

— 2 重積分 —

【1】

下記の (1)〜(8) の 2 重積分を、各小問毎に図で示した領域 D に対して求めよ。

(1) $\displaystyle\iint_{\mathrm{D}} \frac{y}{x}\,dxdy$

(2) $\displaystyle\iint_{\mathrm{D}} x^2y\,dxdy$

(3) $\displaystyle\iint_{\mathrm{D}} \sin(x+y)\,dxdy$

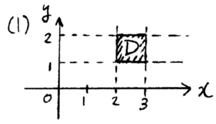
(4) $\displaystyle\iint_{\mathrm{D}} (x+y^3)\,dxdy$

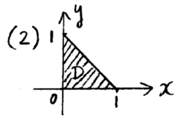
(5) $\displaystyle\iint_{\mathrm{D}} x\,dxdy$

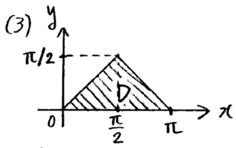
(6) $\displaystyle\iint_{\mathrm{D}} \frac{dxdy}{\sqrt{x^2+y^2}}$

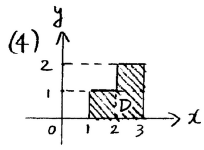
(7) $\displaystyle\iint_{\mathrm{D}} xy^2\,dxdy$

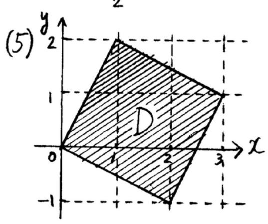
(8) $\displaystyle\iint_{\mathrm{D}} x^2\,dxdy$

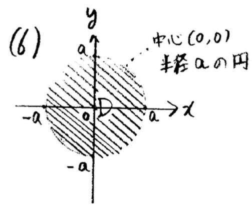


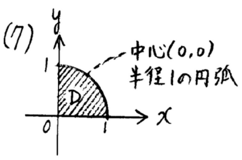


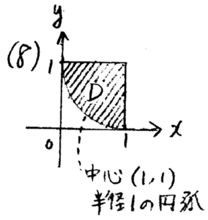












— 累次積分の積分順序の変更 —

【2】

下記の (1)〜(4) の等式の左辺の累次積分の表す積分領域を (x,y) 平面または (r,θ) 平面上に図示し、等式が任意の関数 $f(x,y)$ に対して恒等的に成立するように右辺の積分範囲を表す四角枠内を適切な数や式で埋めよ。

(1) $\displaystyle\int_{-1}^{1} dx \int_0^{1-|x|} dy\, f(x,y) = \int_{\boxed{\text{イ}}}^{\boxed{\text{ア}}} dy \int_{\boxed{\text{エ}}}^{\boxed{\text{ウ}}} dx\, f(x,y)$

(2) $\displaystyle\int_0^1 dx \int_0^{2-x} dy\, f(x,y) = \int_0^1 dy \int_{\boxed{\text{イ}}}^{\boxed{\text{ア}}} dx f(x,y) + \int_1^2 dy \int_{\boxed{\text{エ}}}^{\boxed{\text{ウ}}} dx\, f(x,y)$

(3) $\displaystyle\int_0^1 dx \int_0^2 dy\, f(x,y) + \int_1^2 dx \int_0^1 dy\, f(x,y) + \int_2^3 dx \int_0^2 dy\, f(x,y)$

$\displaystyle= \int_0^1 dy \int_{\boxed{\text{イ}}}^{\boxed{\text{ア}}} dx f(x,y) + \int_1^2 dy \int_{\boxed{\text{エ}}}^{\boxed{\text{ウ}}} dx f(x,y) + \int_{\boxed{\text{カ}}}^{\boxed{\text{オ}}} dy \int_2^3 dx f(x,y)$

$\displaystyle= \int_0^2 dy \int_0^3 dx f(x,y) - \int_{\boxed{\text{ク}}}^{\boxed{\text{キ}}} dy \int_{\boxed{\text{コ}}}^{\boxed{\text{ケ}}} dx f(x,y)$

(4) $\displaystyle\int_0^{\pi/2} d\theta \int_0^{\sin\theta} r dr\, f(r,\theta) = \int_{\boxed{\text{イ}}}^{\boxed{\text{ア}}} r dr \int_{\boxed{\text{エ}}}^{\boxed{\text{ウ}}} d\theta\, f(r,\theta)$