微分積分II　試験問題　（その１）

2002.2.5

学科＿＿＿＿＿＿　番号＿＿＿＿＿＿＿＿　氏名＿＿＿＿＿＿

１．　次の二重積分の値を求めよ。

(1)　$\iint_D (x-y)\,dx\,dy$ ； $D=\{(x,y)\,;\,0\le x\le 1,\ 0\le y\le 2\}$

(2)　$\iint_D y\sin x\,dx\,dy$ ； $D=\{(x,y)\,;\,0\le x\le \frac{\pi}{2},\ 0\le y\le \cos x\}$

(3)　$\iint_D (2x+y)\,dx\,dy$ ； Dは (0,0)　(1,1)　(2,0) を頂点とする三角形の周上および内部である

２．　適当な積分変数の変換を行って，次の二重積分の値を求めよ。

(1)　$\iint_D \log(x^2+y^2)\,dx\,dy$ ； $D=\{(x,y)\,;\,1\le x^2+y^2\le 4\}$

(2)　$\iint_D (x+y)\,e^{x-y}\,dx\,dy$ ； $D=\{(x,y); 0\le x+y\le 2, 0\le x-y\le 2\}$

微分積分Ⅱ　試験問題　（その２）

2002.2.5

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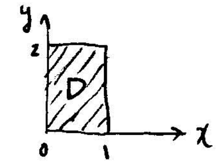
３．　次の二重積分の積分領域を図示し，積分の順序を変更せよ。

(1)　$\int_0^1 dx \int_0^{\sqrt{4x}} f(x,y)dy$　　　　(2)　$\int_{-1}^1 dx \int_0^{e^x} f(x,y)dy$

４．　円柱　$x^2+y^2 \leq a^2$　の　$0 \leq z \leq y$　の部分の体積を求めよ。

５．　半径　a　の球の表面積は　$4\pi a^2$　に等しいことを，積分を用いて表面積を求める方法により導け。

1、(1)

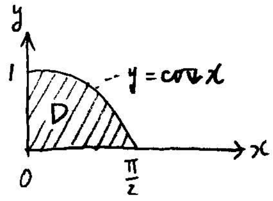


$$I=\iint_D (x-y)\,dx\,dy=\int_0^1 dx\int_0^2 dy\,(x-y)$$

$$=\int_0^1 dx\left[xy-\frac{1}{2}y^2\right]_{y=0}^{y=2}=\int_0^1 dx\,(2x-2)$$

$$=\left[x^2-2x\right]_{x=0}^{x=1}=1-2=\underline{-1}$$

(2)

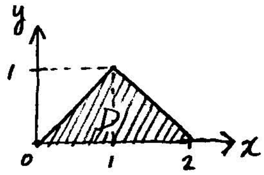


$$I=\iint_D y\sin x\,dx\,dy=\int_0^{\pi/2} dx\int_0^{\cos x} dy\;y\sin x$$

$$=\int_0^{\pi/2} dx\,\sin x\left[\frac{1}{2}y^2\right]_0^{\cos x}=\frac{1}{2}\int_0^{\pi/2} dx\,\sin x\cos^2 x$$

$$=\frac{1}{2}\left[-\frac{1}{3}\cos^3 x\right]_{x=0}^{x=\pi/2}=\frac{1}{2}\left(-0+\frac{1}{3}\right)=\underline{\frac{1}{6}}$$

(3)



$$I=\iint_D (2x+y)\,dx\,dy=\int_0^1 dx\int_0^x dy\,(2x+y)+\int_1^2 dx\int_0^{2-x} dy\,(2x+y)$$

$$=\int_0^1 dx\left[2xy+\frac{1}{2}y^2\right]_{y=0}^{y=x}+\int_1^2 dx\left[2xy+\frac{1}{2}y^2\right]_{y=0}^{y=2-x}$$

$$=\int_0^1 dx\left(2x^2+\frac{1}{2}x^2\right)+\int_1^2 dx\left\{2x(2-x)+\frac{1}{2}(2-x)^2\right\}$$

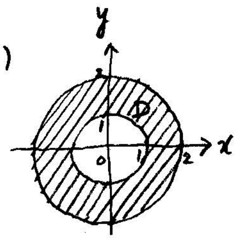
$$=\frac{5}{2}\int_0^1 dx\,x^2+\int_1^2 dx\left(4x-2x^2+2-2x+\frac{1}{2}x^2\right)$$

$$=\frac{5}{2}\left[\frac{1}{3}x^3\right]_{x=0}^{x=1}+\int_1^2 dx\left(-\frac{3}{2}x^2+2x+2\right)$$

$$=\frac{5}{2}\frac{1}{3}+\left[-\frac{1}{2}x^3+x^2+2x\right]_{x=1}^{x=2}=\frac{5}{6}+\left(-6+4+2+\frac{3}{2}-2-2\right)$$

$$=\frac{5}{6}-4+\frac{3}{2}=\frac{5-24+9}{6}=-\frac{10}{6}=\underline{-\frac{5}{3}}$$

2. (1)

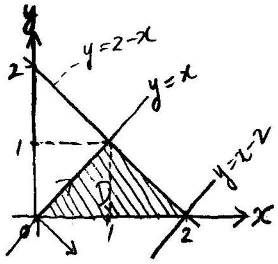


$$I=\iint_D \log(x^2+y^2)\,dx\,dy = \int_1^2 r\,dr\int_0^{2\pi}d\theta\,\log r^2$$

$$=2\pi\frac{1}{2}\int_{r=1}^{r=2}\log r^2\,d(r^2) = \pi\left[r^2\log r^2-r^2\right]_{r=1}^{r=2}$$

$$=\pi\left(4\log 2^2-4-1\log 1^2+1^2\right)=\underline{\pi\left(8\log 2-3\right)}$$

(2)



$$I=\iint_D (x+y)\,e^{x-y}\,dx\,dy$$

$\begin{pmatrix}u\\v\end{pmatrix}=\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$, $\begin{pmatrix}x\\y\end{pmatrix}=-\frac{1}{2}\begin{pmatrix}-1&-1\\-1&1\end{pmatrix}\begin{pmatrix}u\\v\end{pmatrix}$ とおくと.

$$J=\frac{\partial(x,y)}{\partial(u,v)}=\left(-\frac{1}{2}\right)^2(-1-1)=-\frac{1}{2}$$

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}\frac{1}{2}\\\frac{1}{2}\end{pmatrix}u+\begin{pmatrix}\frac{1}{2}\\-\frac{1}{2}\end{pmatrix}v$$

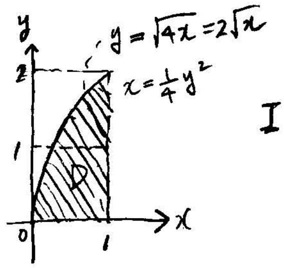
$$I=\int_0^2 dv\int_v^2 du\,|J|\,u\,e^v=\frac{1}{2}\int_0^2 dv\,e^v\left[\frac{1}{2}u^2\right]_{u=v}^{u=2}$$

$$=\frac{1}{2}\int_0^2 dv\;e^v\frac{1}{2}\left(4-v^2\right)=\frac{1}{4}\left[4e^v-e^v(v^2-2v+2)\right]_{v=0}^{v=2}$$

$$\left(\because \int e^v v^2\,dv=e^v v^2-\int e^v\,2v\,dv=e^v v^2-2ve^v+2\int e^v\,dv=e^v\left(v^2-2v+2\right)+C\right)$$

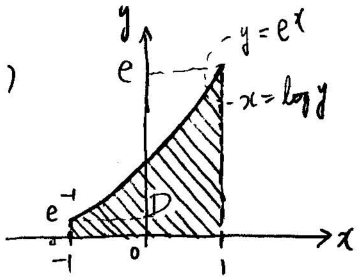
$$I=\frac{1}{4}\left\{e^2\left(4-4+4-2\right)\right\}=\frac{1}{2}e^2$$

3、(1)



$$I=\int_0^1 dx\int_0^{\sqrt{4x}} f(x,y)\,dy=\int_0^2 dy\int_{y^2/4}^{1} f(x,y)\,dx$$

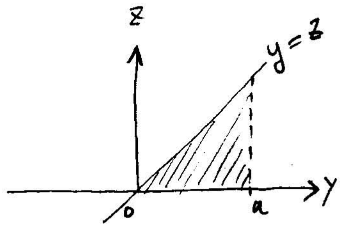
(2)

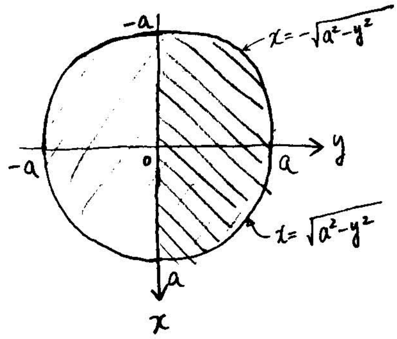


$$I=\int_{-1}^1 dx\int_0^{e^x} f(x,y)\,dy=\int_0^{1/e} dy\int_{-1}^{1} f(x,y)\,dx$$

$$+\int_{1/e}^{e} dy\int_{\log y}^{1} f(x,y)\,dx$$

4.





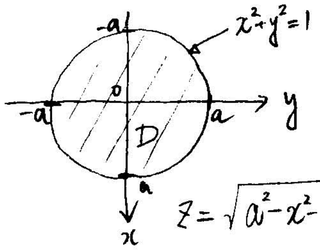
$$V=\int_0^a dy\int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} dx\int_0^y dz$$

$$=\int_0^a dy\ 2\sqrt{a^2-y^2}\ y$$

$$=2\left[-\frac{1}{3}(a^2-y^2)^{3/2}\right]_{y=0}^{y=a}$$

$$=2\left(0+\frac{1}{3}a^3\right)=\frac{2}{3}a^3$$

5、



$z=\sqrt{a^2-x^2-y^2}$

$$\frac{\partial z}{\partial x} = -x(a^2-x^2-y^2)^{-1/2}$$

$$\frac{\partial z}{\partial y} = -y(a^2-x^2-y^2)^{-1/2}$$

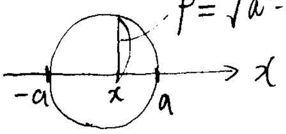
$$\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2} = \sqrt{1+(x^2+y^2)(a^2-x^2-y^2)^{-1}} = \sqrt{\frac{a^2}{a^2-x^2-y^2}}$$

$$\frac{1}{2}S = \iint_D \frac{a}{\sqrt{a^2-x^2-y^2}}\,dxdy$$

$$= \int_0^{2\pi} d\theta \int_0^a r\,dr\,\frac{a}{\sqrt{a^2-r^2}} = 2\pi a\left[-\sqrt{a^2-r^2}\right]_{r=0}^{r=a} = 2\pi a(0+\sqrt{a^2})$$

$$= 2\pi a^2 \qquad \therefore\ S = 4\pi a^2$$

又は



$$\sqrt{1+P'^2} = \sqrt{1+\left(\frac{-x}{\sqrt{a^2-x^2}}\right)^2} = \sqrt{\frac{a^2}{a^2-x^2}}$$

$$S = \int_{-a}^{a} 2\pi P\sqrt{1+P'^2}\,dP = 2\pi\int_{-a}^{a}\sqrt{a^2-x^2}\sqrt{\frac{a^2}{a^2-x^2}}\,dx = 2\pi\int_{-a}^{a} a\,dx$$

$$= 2\pi a\cdot 2a = 4\pi a^2$$