テーラー展開のグラフ　：　$y = f(x) = (1+x)^{1/2}$ の場合

$f(x) = (1+x)^{1/2}$

$f_n(x) = \displaystyle\sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k$　：　$f(x)$ のテーラー展開（級数）の x の べき が 0 乗から n 乗までの項の和。

具体的な形は：

$f_1(x) = 1 + \frac{1}{2}x$

$f_2(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2$

$f_3(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$

$f_4(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4$

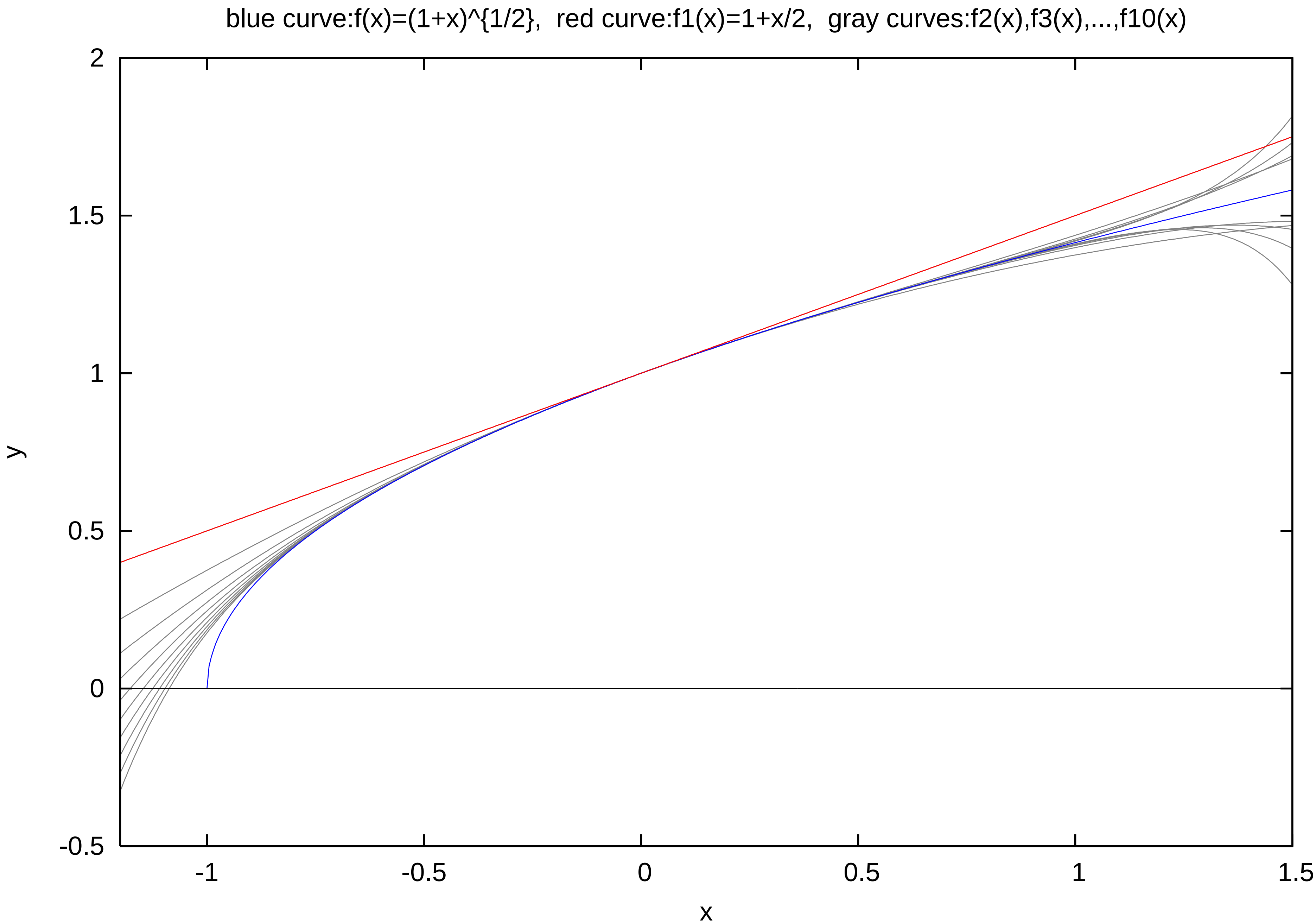
$f_5(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \frac{7}{256}x^5$

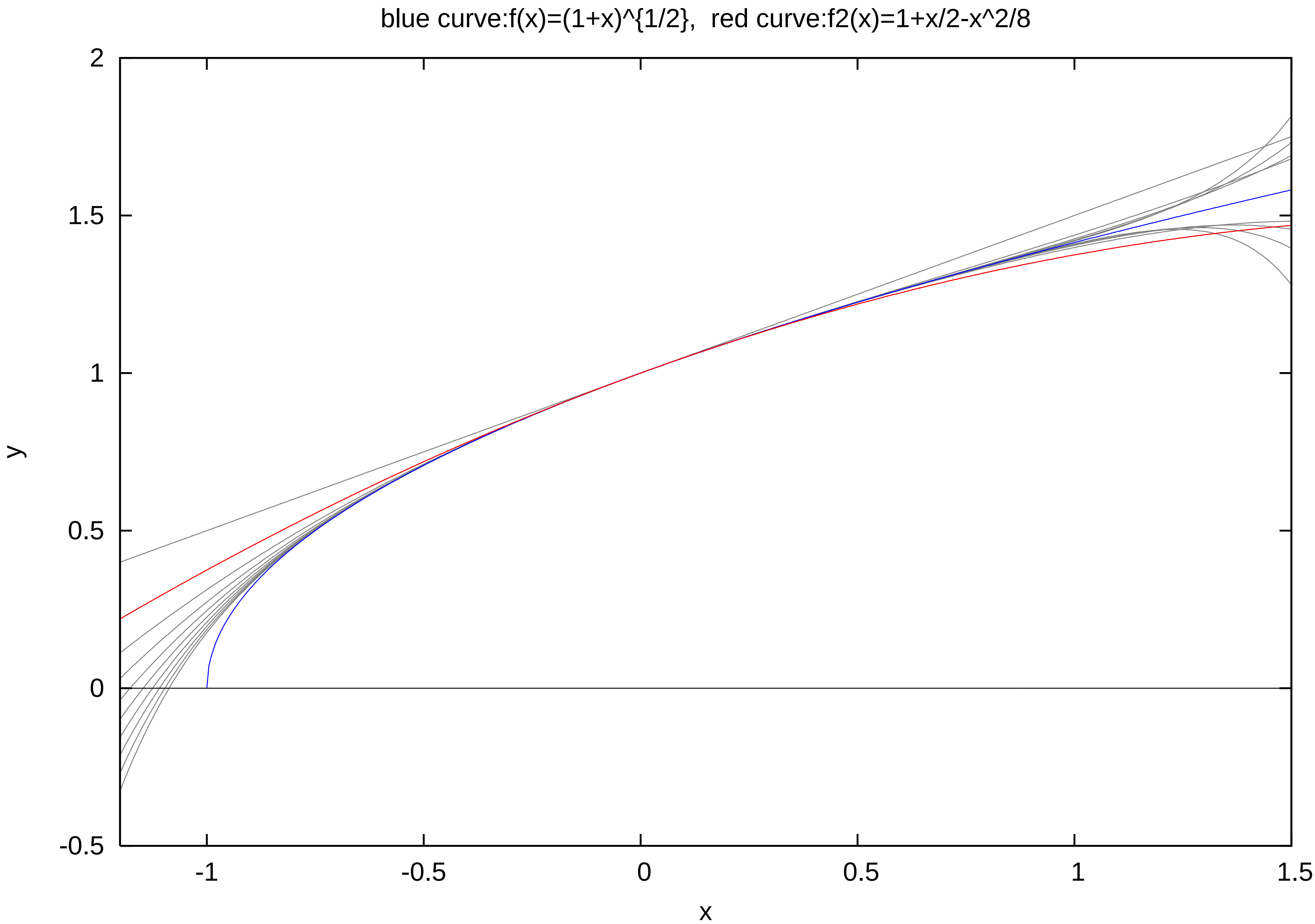
$f_6(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \frac{7}{256}x^5 - \frac{21}{1024}x^6$

$f_7(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \frac{7}{256}x^5 - \frac{21}{1024}x^6 + \frac{33}{2048}x^7$

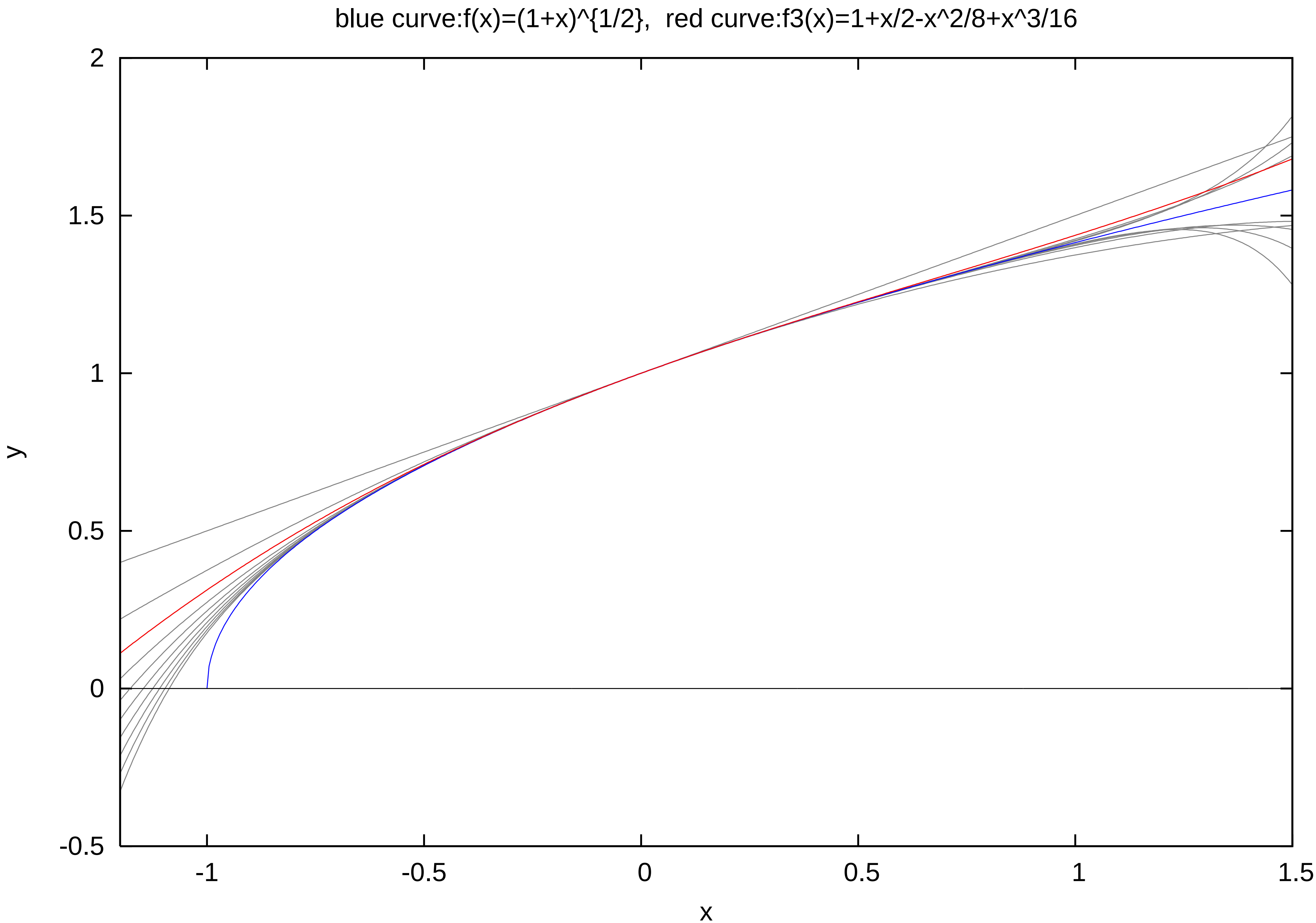
$\cdots$

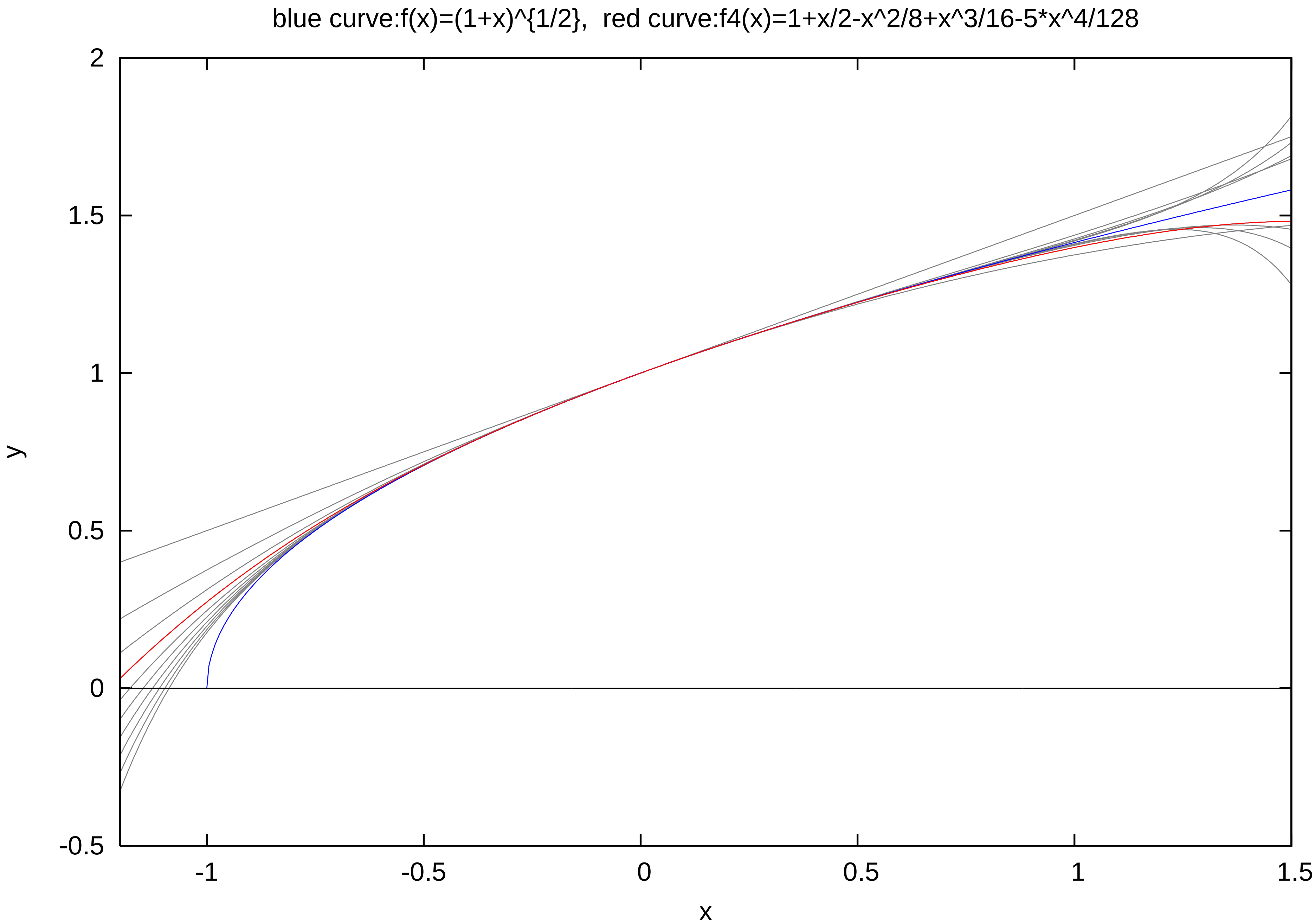
【注】$(1+x)^{1/2}$ のテーラー展開の収束半径は 1 である。即ち、$|x| < 1$ で $f_n(x) \to f(x)$ $(n \to \infty)$ となるが、$|x| > 1$ では $f_n(x)$ は $n \to \infty$ で発散する。


blue curve:f(x)=(1+x)^{1/2}, red curve:f1(x)=1+x/2, gray curves:f2(x),f3(x),...,f10(x)
2
1.5
1
0.5
0
-0.5
y
-1
-0.5
0
0.5
1
1.5
x

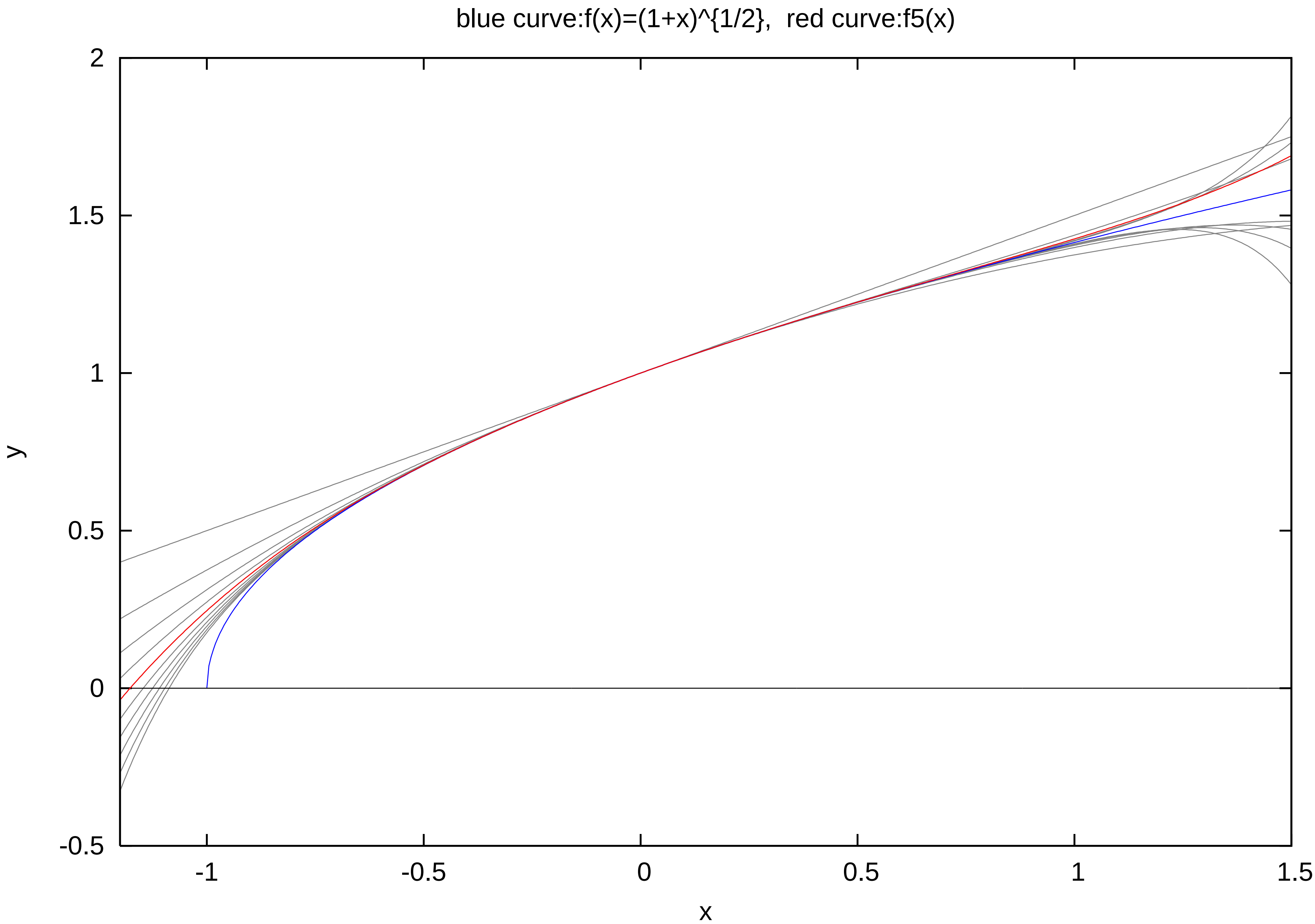


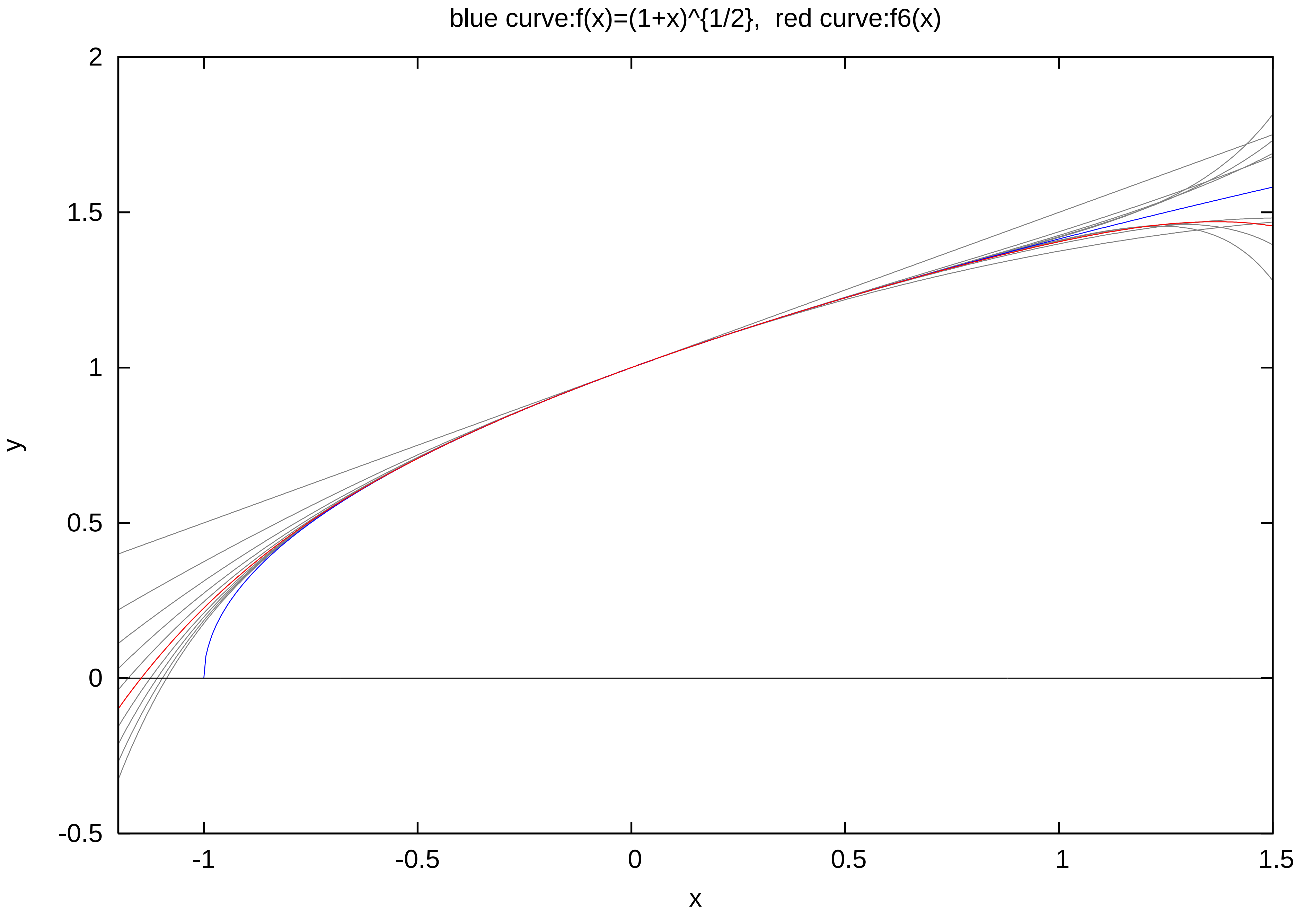
blue curve:f(x)=(1+x)^{1/2}, red curve:f2(x)=1+x/2-x^2/8
2
1.5
1
0.5
0
-0.5
y
-1
-0.5
0
0.5
1
1.5
x

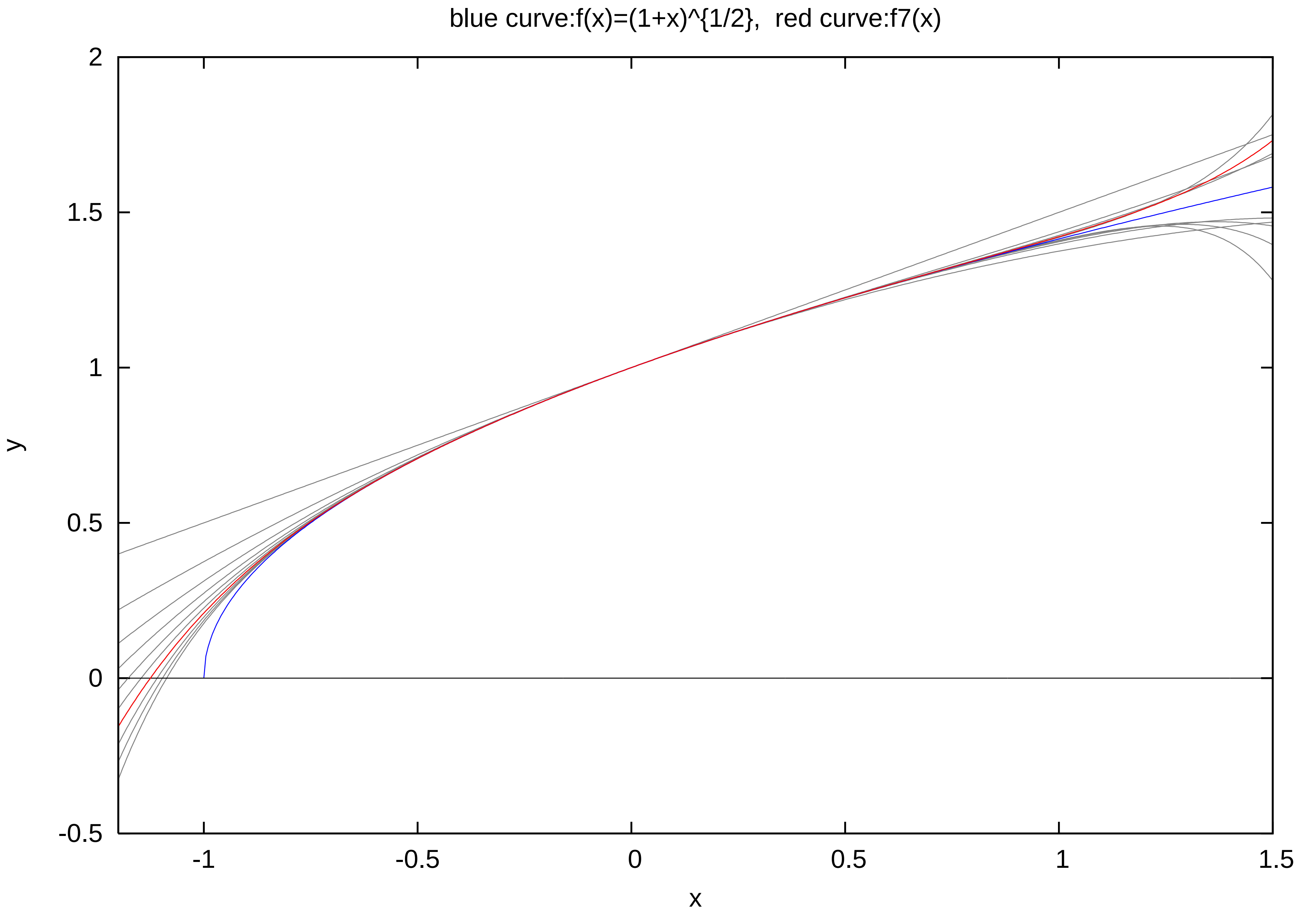

blue curve:f(x)=(1+x)^{1/2}, red curve:f3(x)=1+x/2-x^2/8+x^3/16
y
x
2
1.5
1
0.5
0
-0.5
-1
-0.5
0
0.5
1
1.5

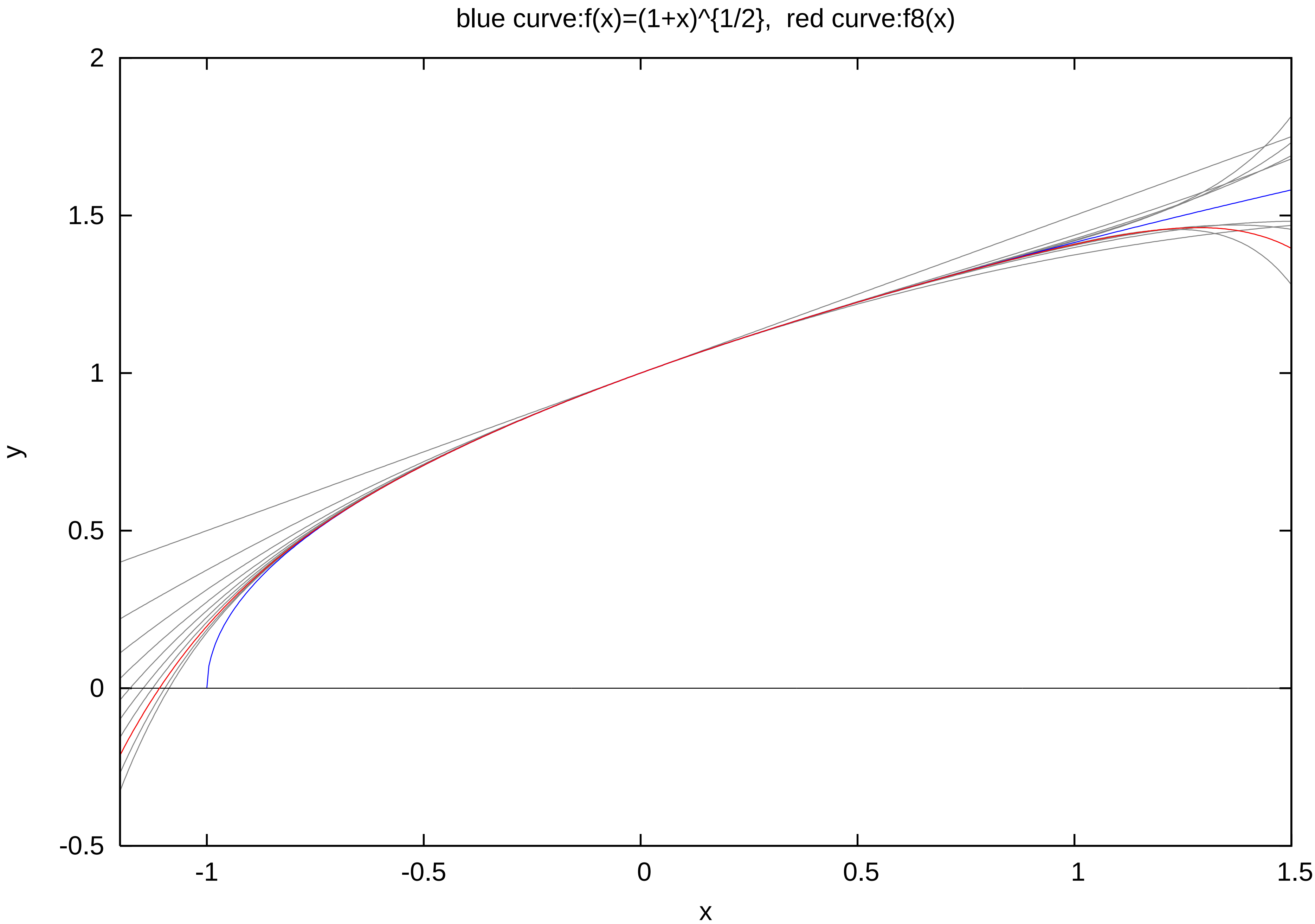


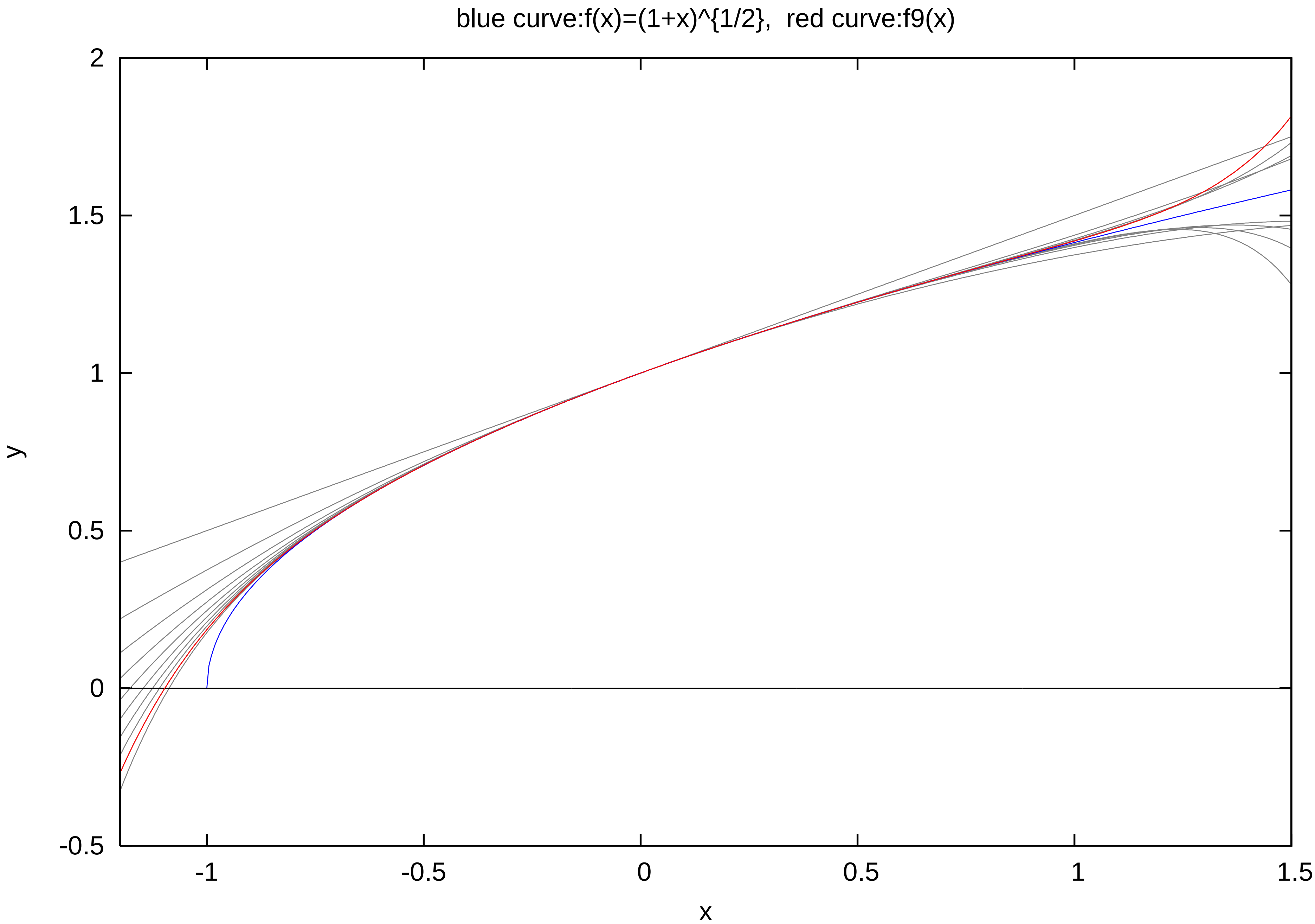
blue curve:f(x)=(1+x)^{1/2}, red curve:f4(x)=1+x/2-x^2/8+x^3/16-5*x^4/128
2
1.5
1
0.5
0
-0.5
y
-1
-0.5
0
0.5
1
1.5
x

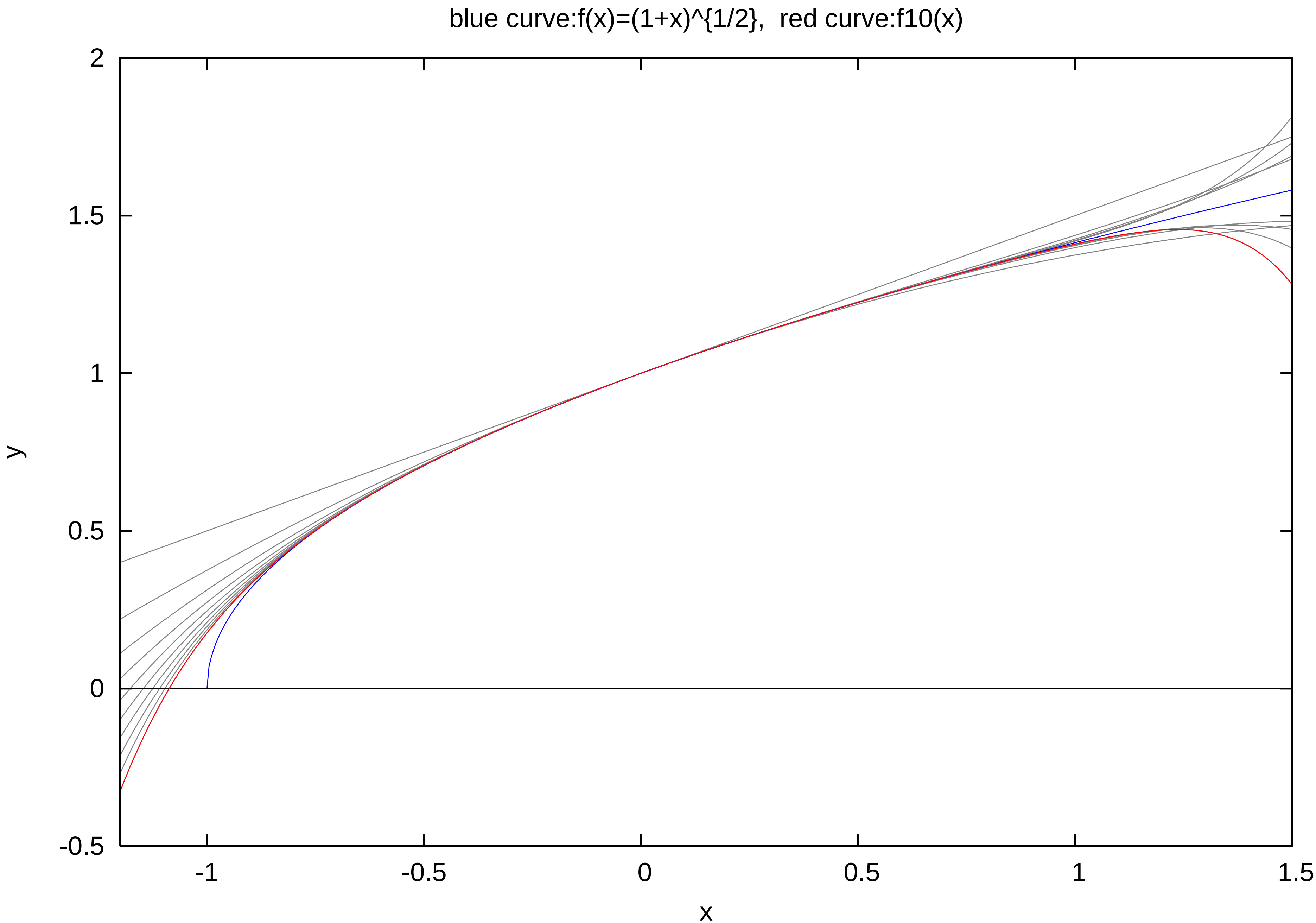

blue curve:f(x)=(1+x)^{1/2}, red curve:f5(x)
y
x
2
1.5
1
0.5
0
-0.5
-1
-0.5
0
0.5
1
1.5


blue curve:f(x)=(1+x)^{1/2}, red curve:f6(x)
y
x
2
1.5
1
0.5
0
-0.5
-1
-0.5
0
0.5
1
1.5

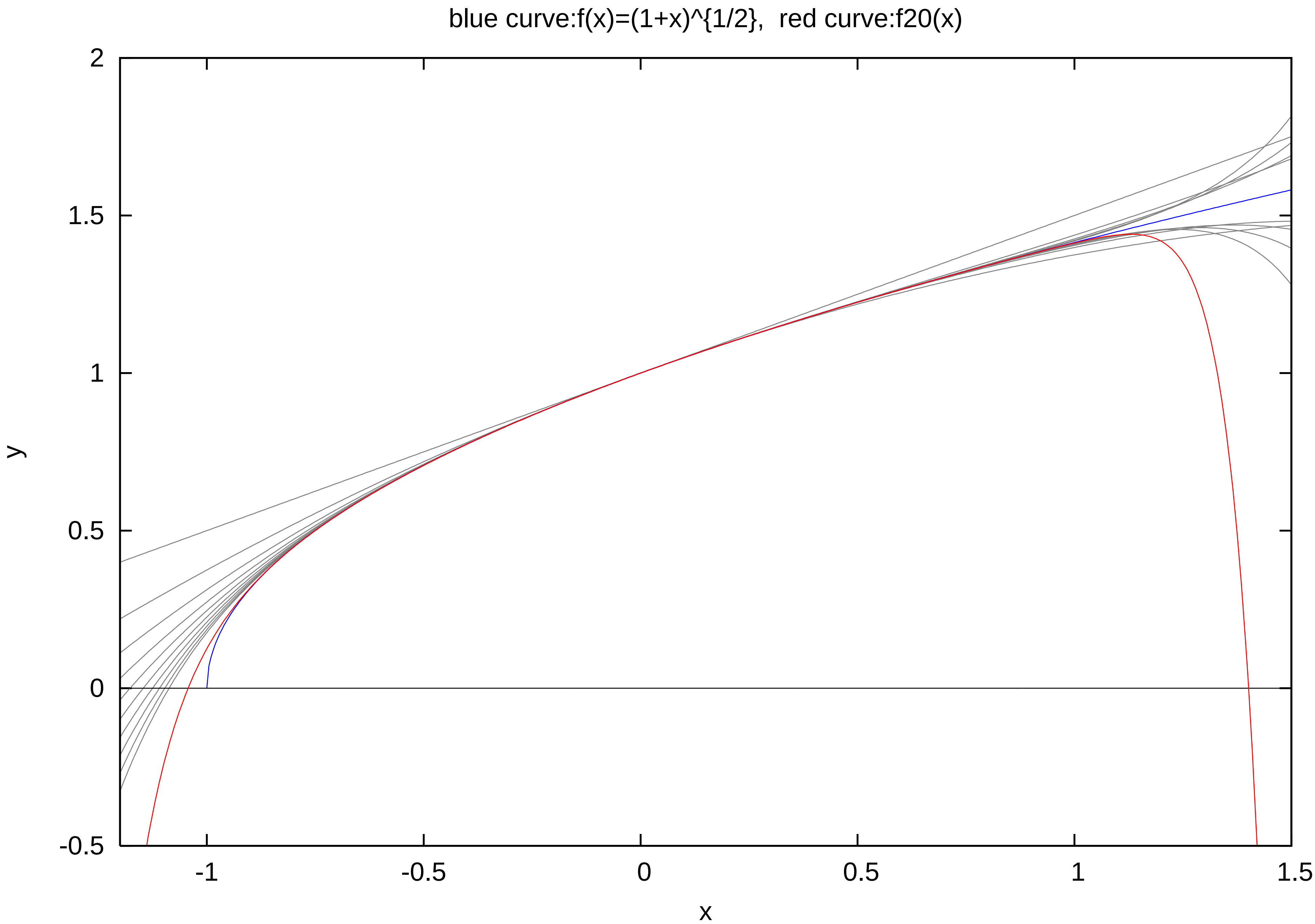

blue curve:f(x)=(1+x)^{1/2}, red curve:f7(x)
y
x
2
1.5
1
0.5
0
-0.5
-1
-0.5
0
0.5
1
1.5


blue curve:f(x)=(1+x)^{1/2}, red curve:f8(x)
y
x
2
1.5
1
0.5
0
-0.5
-1
-0.5
0
0.5
1
1.5

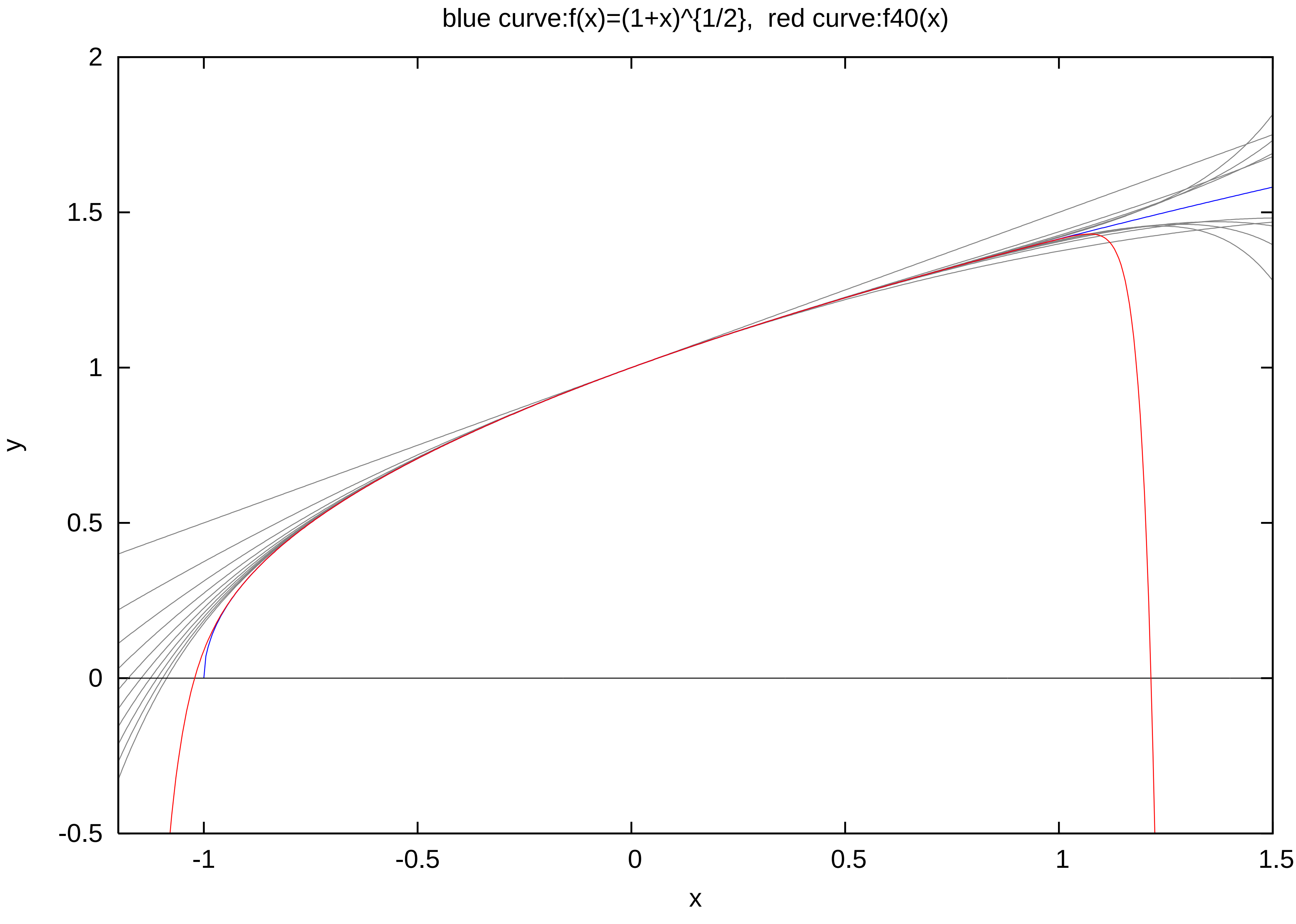

blue curve:f(x)=(1+x)^{1/2}, red curve:f9(x)
y
x
2
1.5
1
0.5
0
-0.5
-1
-0.5
0
0.5
1
1.5

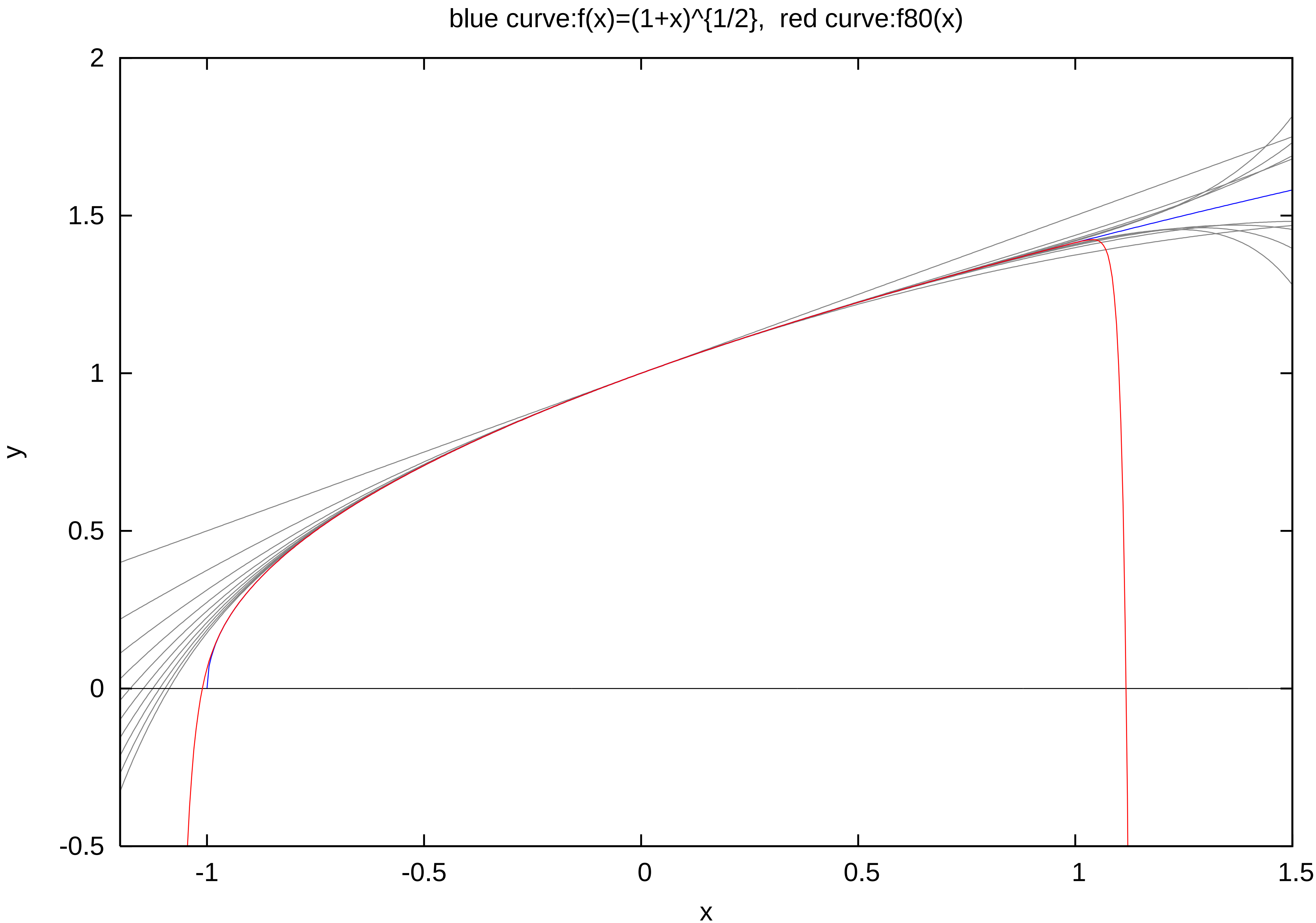


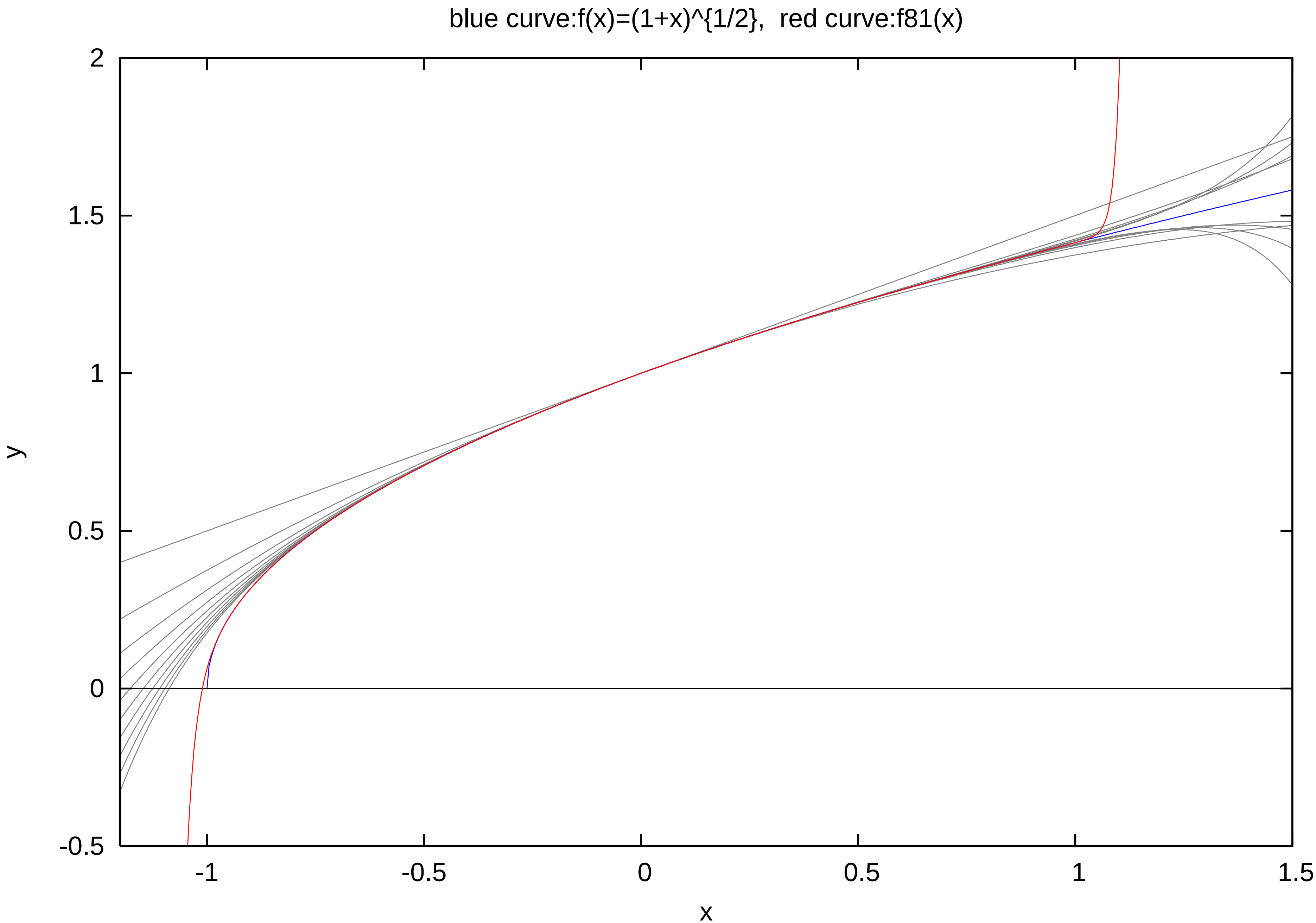
blue curve:f(x)=(1+x)^{1/2}, red curve:f10(x)
y
x
2
1.5
1
0.5
0
-0.5
-1
-0.5
0
0.5
1
1.5



blue curve:f(x)=(1+x)^{1/2}, red curve:f20(x)
2
1.5
1
0.5
0
-0.5
y
-1
-0.5
0
0.5
1
1.5
x


blue curve:f(x)=(1+x)^{1/2}, red curve:f40(x)
2
1.5
1
0.5
0
-0.5
y
-1
-0.5
0
0.5
1
1.5
x


blue curve:f(x)=(1+x)^{1/2}, red curve:f80(x)
2
1.5
1
0.5
0
-0.5
-1
-0.5
0
0.5
1
1.5
x
y


blue curve:f(x)=(1+x)^{1/2}, red curve:f81(x)
y
x
2
1.5
1
0.5
0
-0.5
-1
-0.5
0
0.5
1
1.5