

テーラー展開のグラフ : $y = f(x) = \sin x$ の場合

$$f(x) = \sin x$$

$$f_n(x) = \sum_{k=0}^n \frac{f^{(k)}(0)}{k!} x^k \quad : \quad f(x) \text{ のテーラー展開 (級数) の } x \text{ の べき が } 0 \text{ 乗から } n \text{ 乗までの項の和。}$$

具体的な形は :

$$f_1(x) = x$$

$$f_3(x) = x - \frac{1}{6}x^3$$

$$f_5(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5$$

$$f_7(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7$$

$$f_9(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \frac{1}{362880}x^9$$

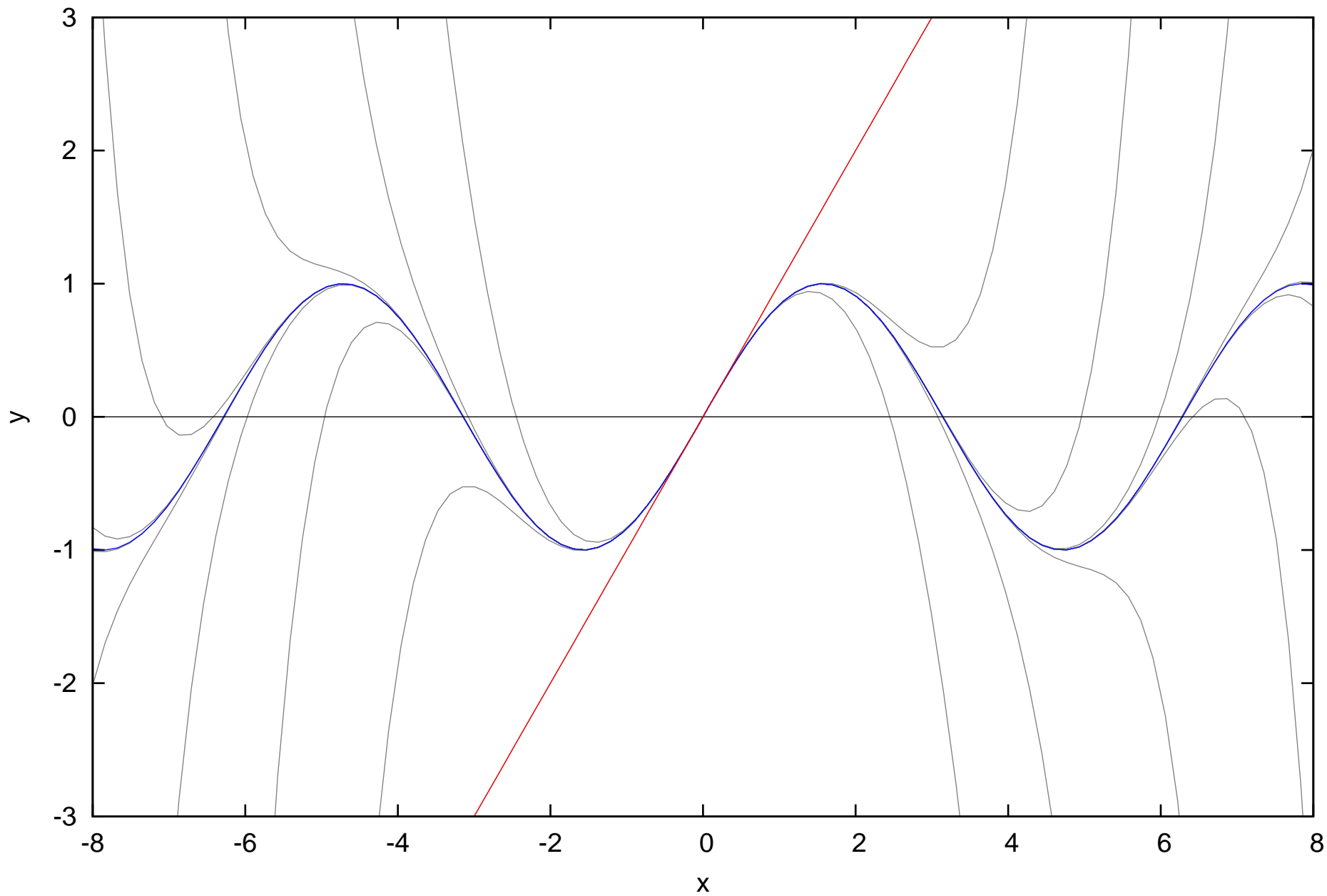
$$f_{11}(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \frac{1}{362880}x^9 - \frac{1}{3991680}x^{11}$$

$$f_{13}(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \frac{1}{362880}x^9 - \frac{1}{3991680}x^{11} + \frac{1}{6227020800}x^{13}$$

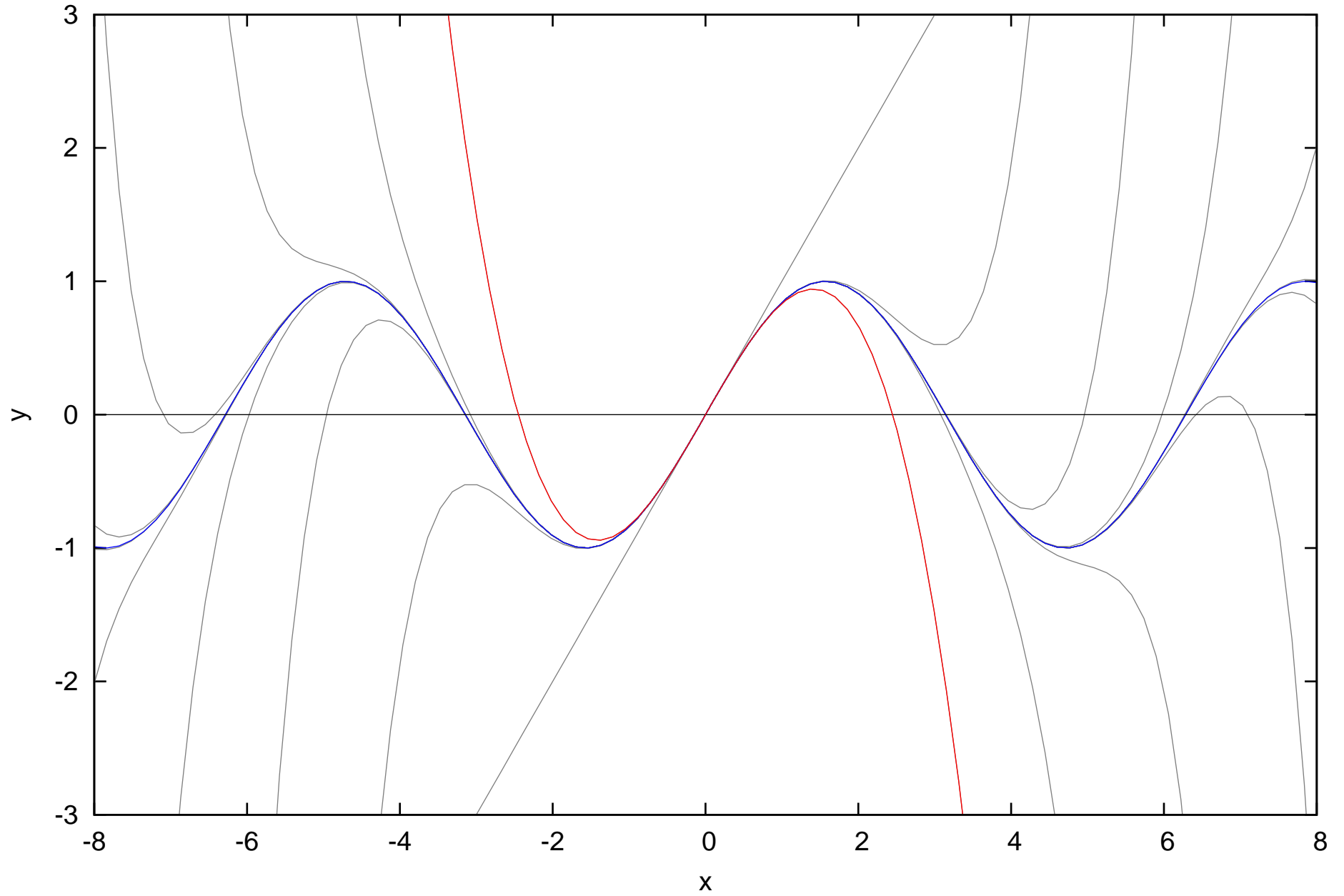
...

【注】 $\sin x$ のように奇関数のテーラー展開では、 x の偶数べきの項の係数はゼロとなる。したがって、 $f_2(x) = f_1(x)$, $f_4(x) = f_3(x)$, $f_6(x) = f_5(x)$, ... である。

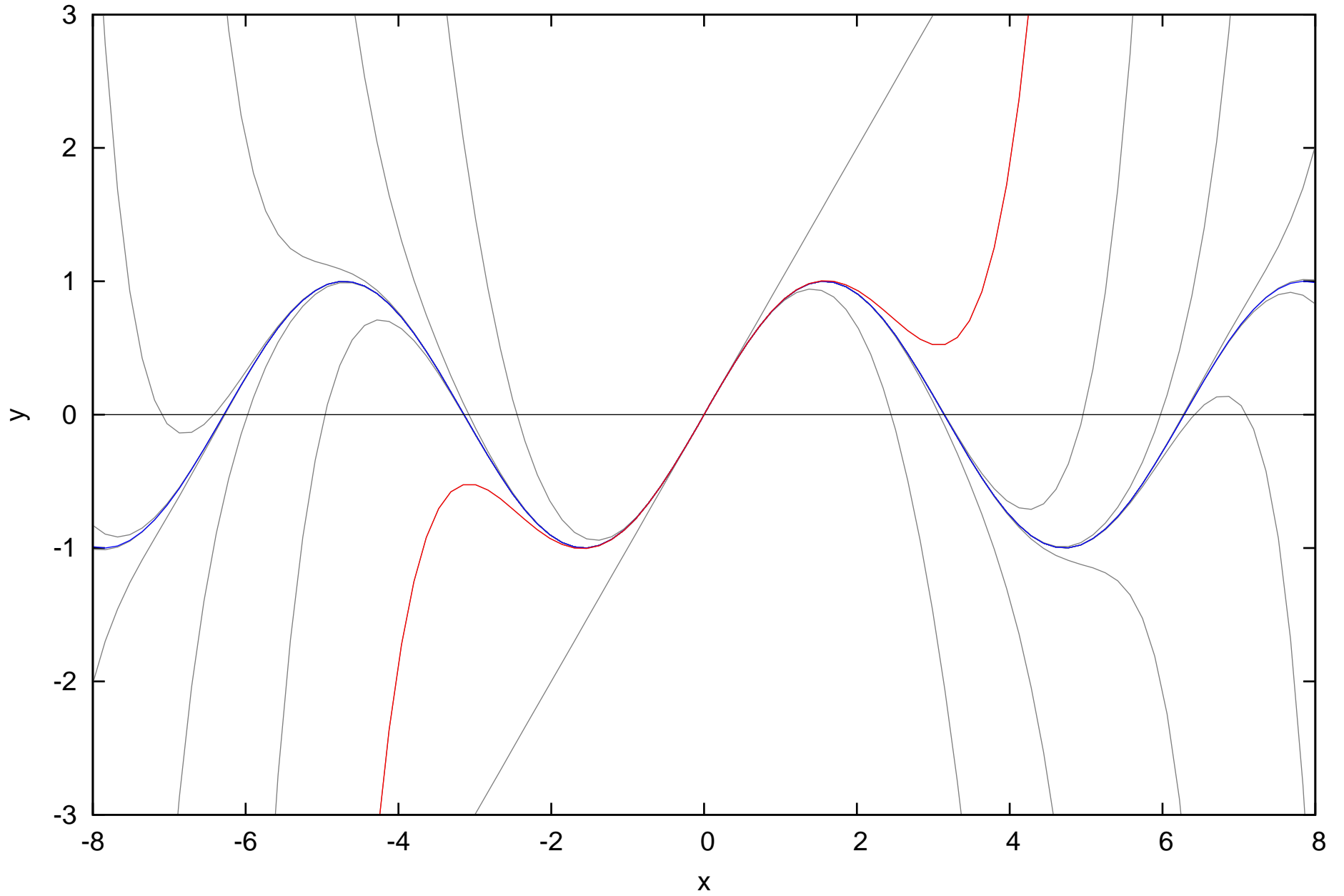
blue curve: $f(x)=\sin(x)$, red curve: $f_1(x)=x$, gray curves: $f_3(x), f_5(x), \dots, f_{21}(x)$



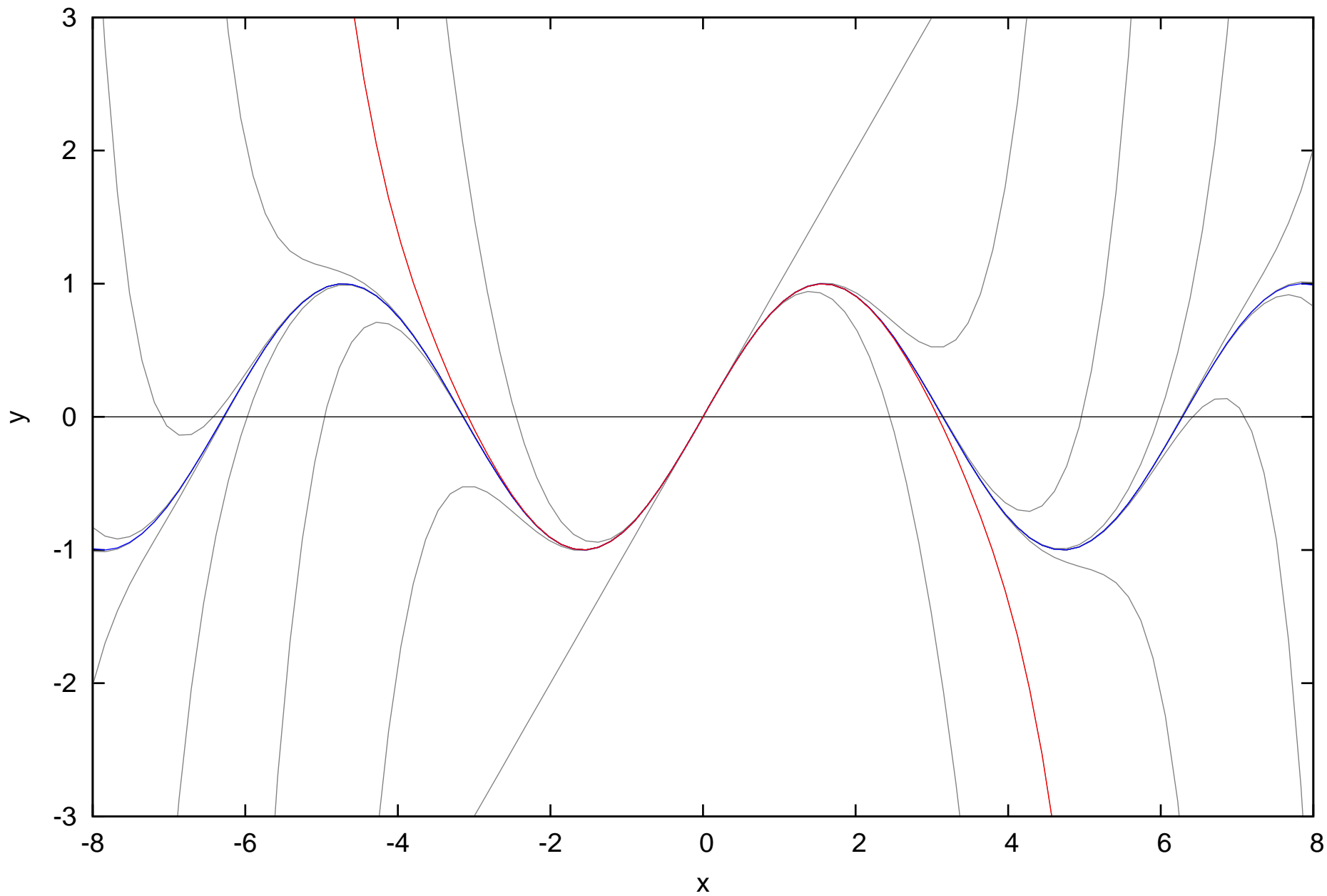
blue curve: $f(x)=\sin(x)$, red curve: $f_3(x)=x-x^3/6$



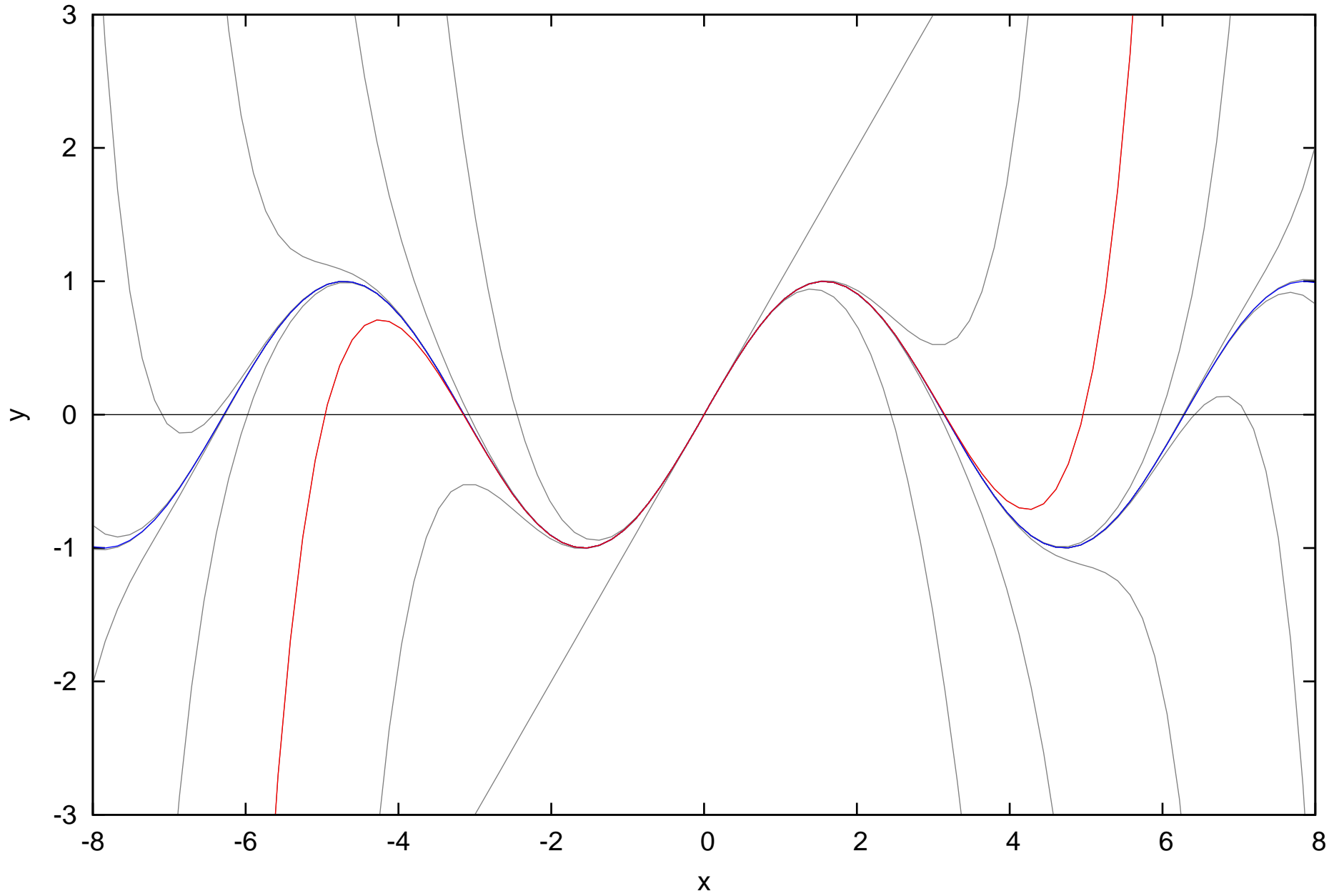
blue curve: $f(x)=\sin(x)$, red curve: $f_5(x)=x-x^3/6+x^5/120$



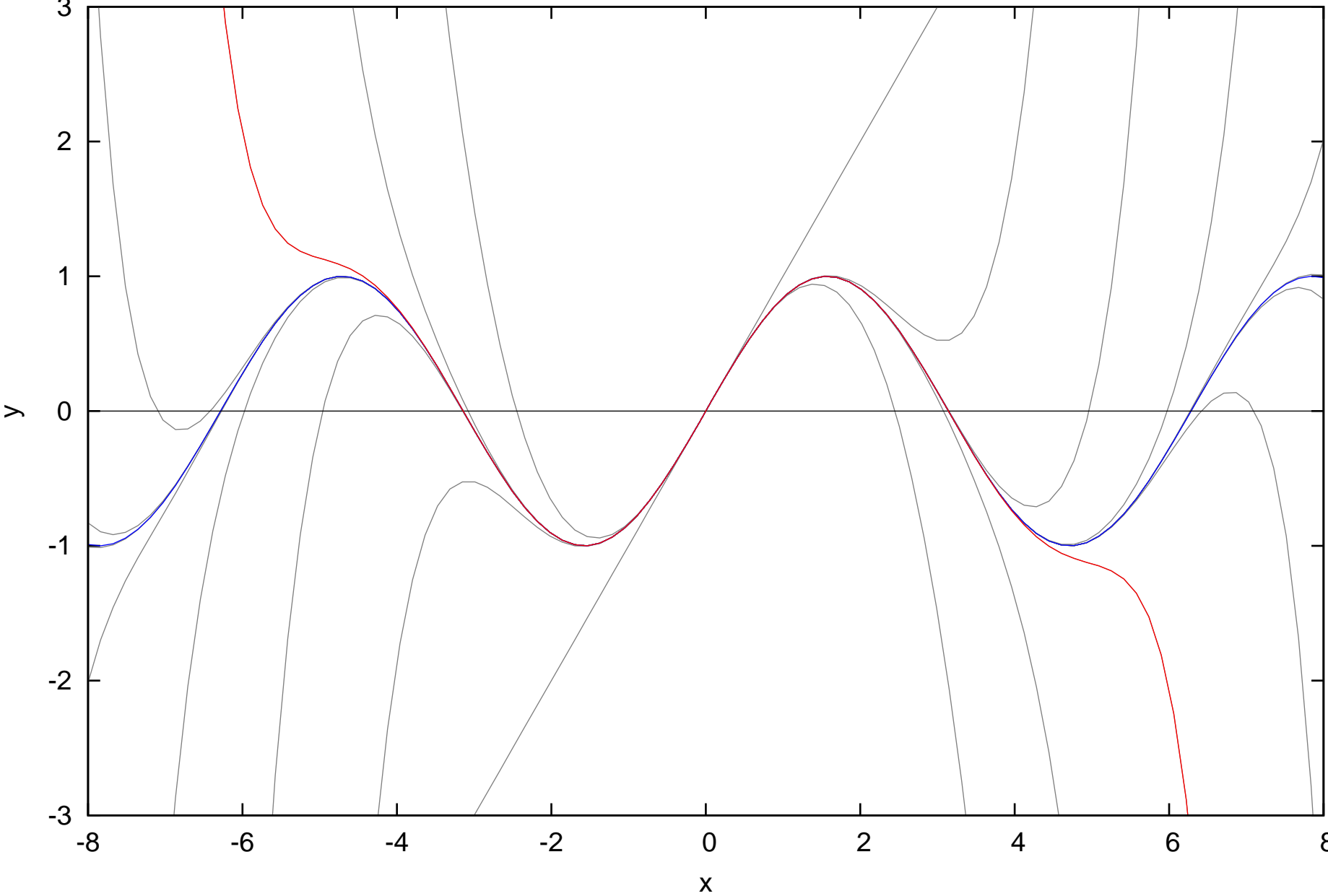
blue curve: $f(x)=\sin(x)$, red curve: $f7(x)=x-x^3/6+x^5/120-x^7/5040$



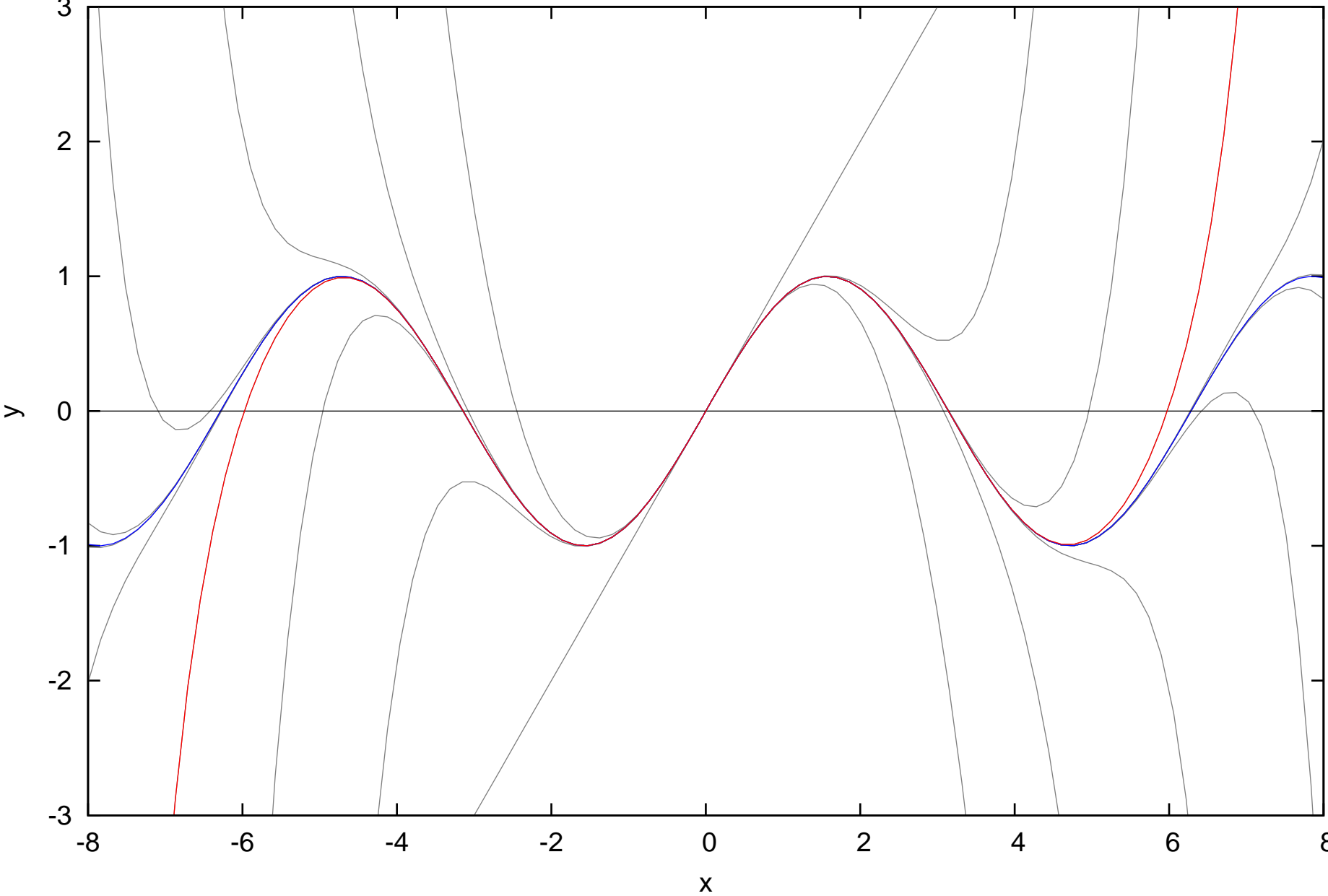
blue curve: $f(x)=\sin(x)$, red curve: $f_9(x)$



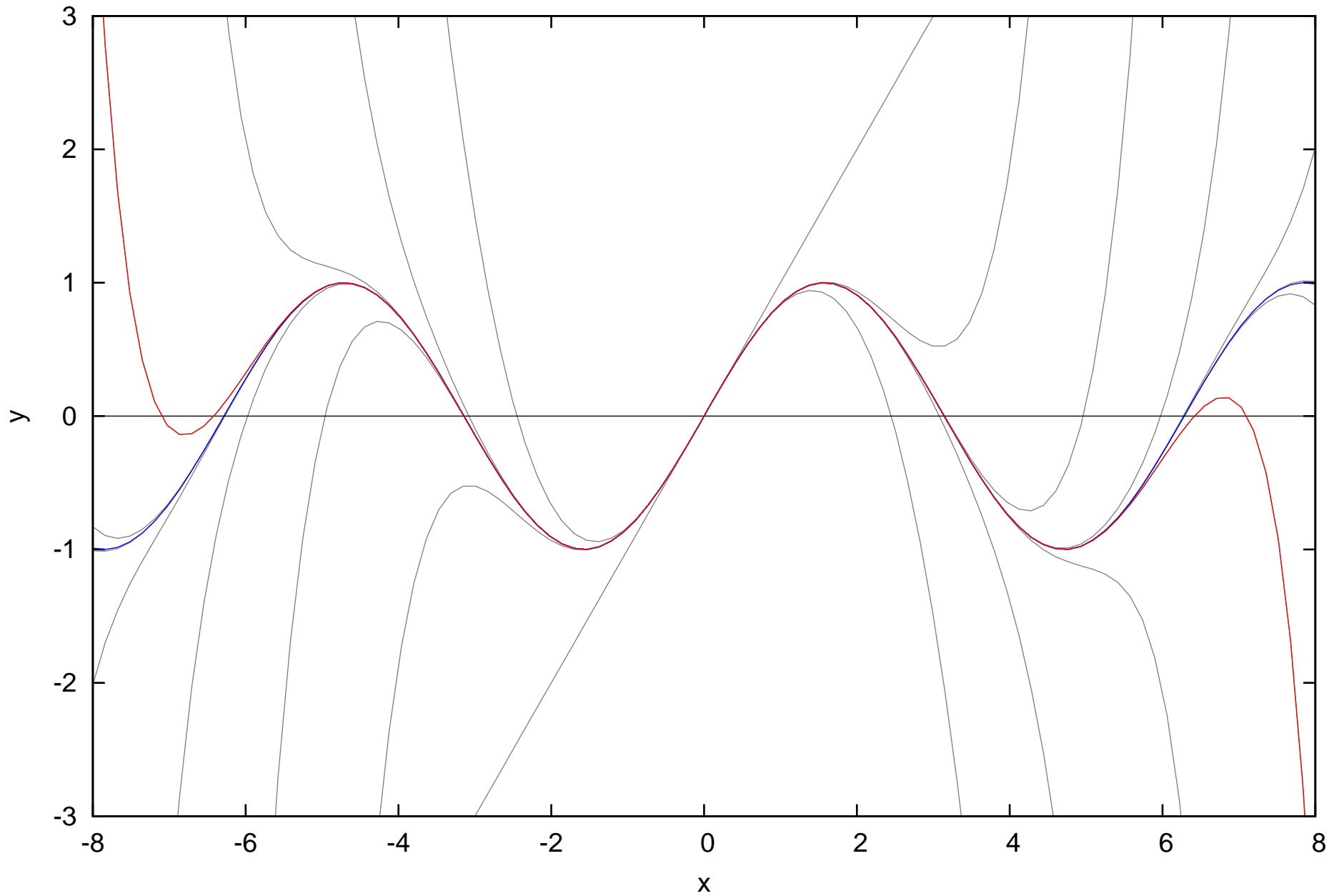
blue curve: $f(x)=\sin(x)$, red curve: $f_{11}(x)$



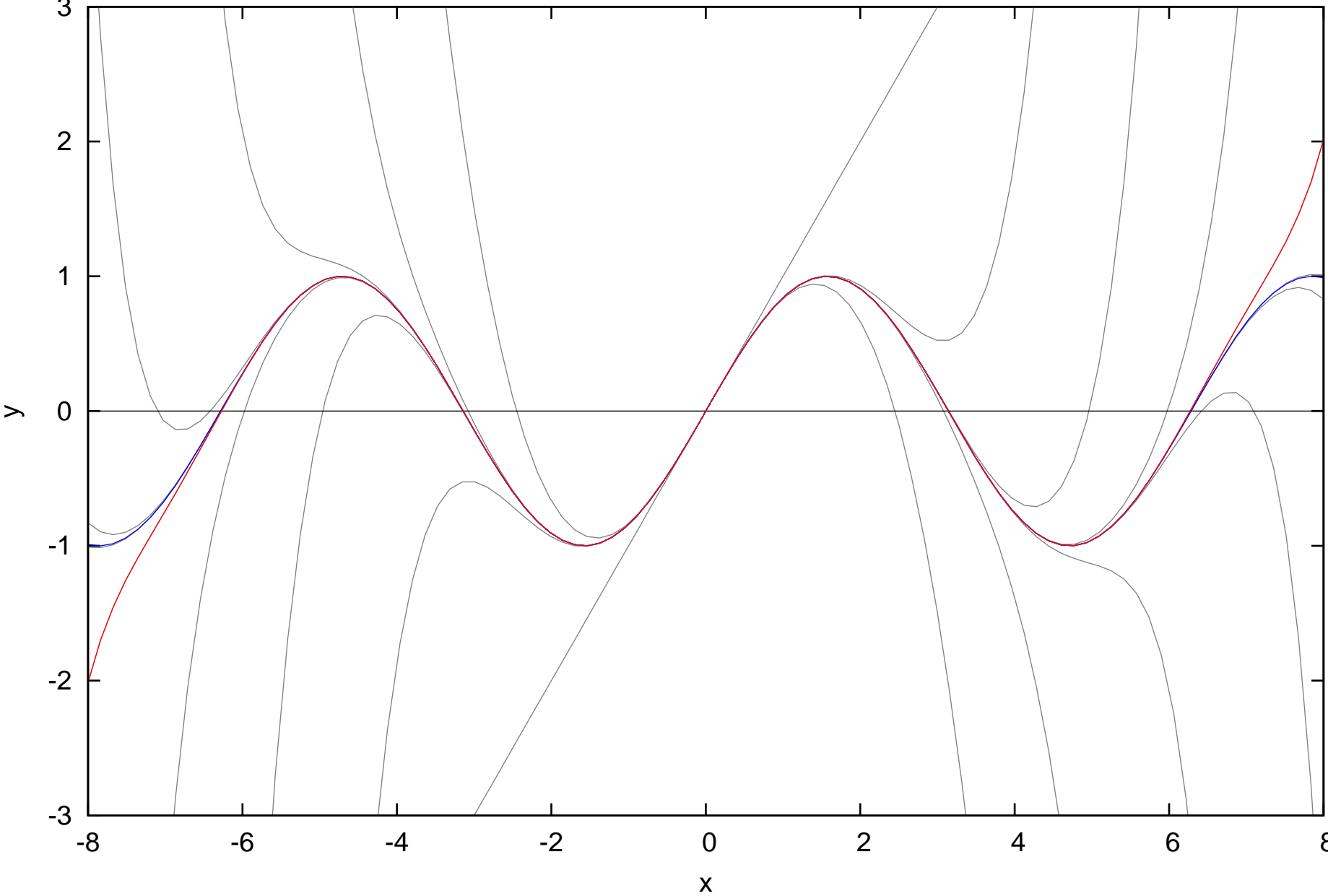
blue curve: $f(x)=\sin(x)$, red curve: $f_{13}(x)$



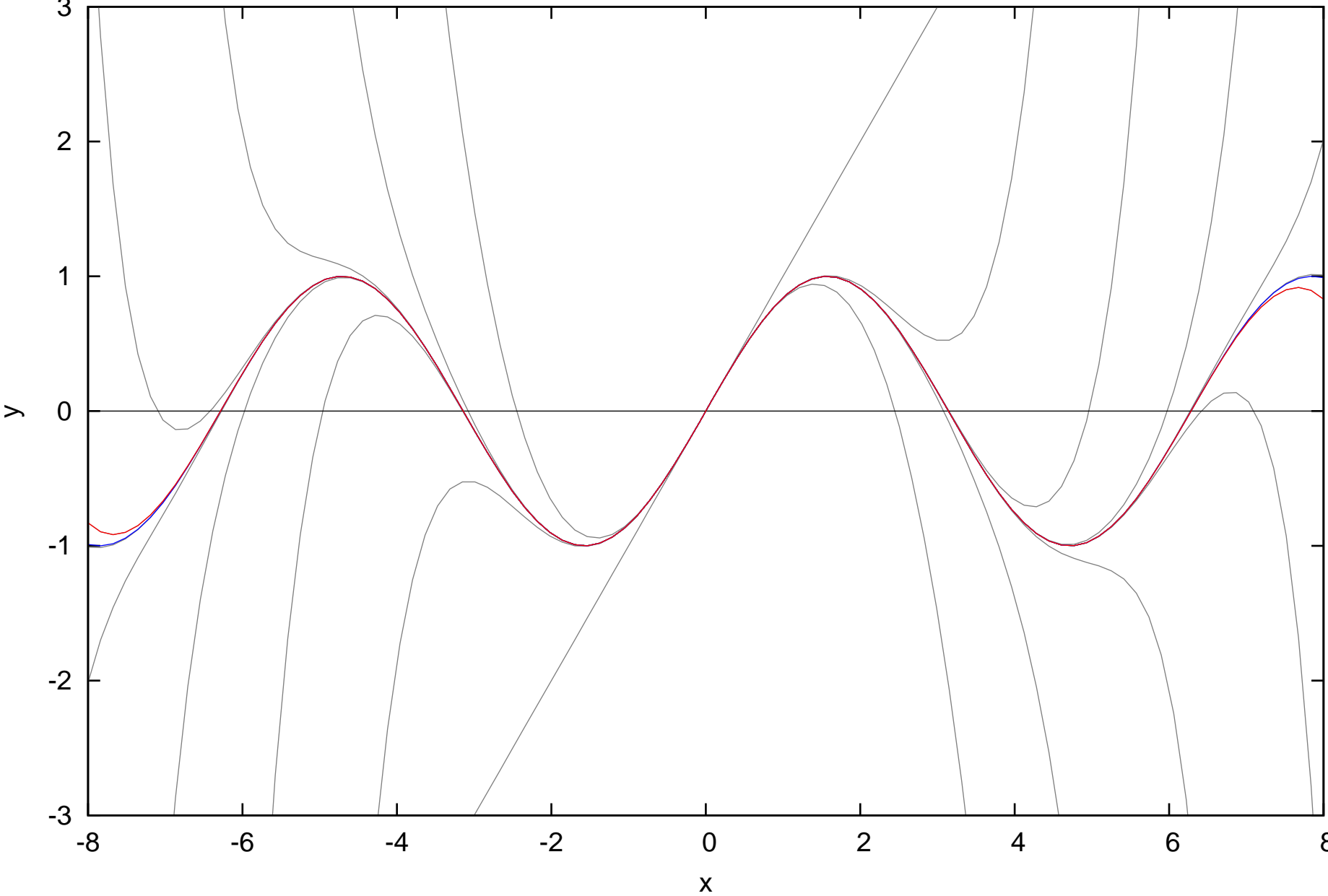
blue curve: $f(x)=\sin(x)$, red curve: $f_{15}(x)$



blue curve: $f(x)=\sin(x)$, red curve: $f_{17}(x)$



blue curve: $f(x)=\sin(x)$, red curve: $f_{19}(x)$



blue curve: $f(x)=\sin(x)$, red curve: $f_{21}(x)$

