

テーラー展開のグラフ : $y = f(x) = \log(1+x)$ の場合

$$f(x) = \log(1+x)$$

$$f_n(x) = \sum_{k=0}^n \frac{f^{(k)}(0)}{k!} x^k \quad : \quad f(x) \text{ のテーラー展開 (級数) の } x \text{ のべきが } 0 \text{ 乗から } n \text{ 乗までの項の和。}$$

具体的な形は :

$$f_1(x) = x$$

$$f_2(x) = x - \frac{1}{2}x^2$$

$$f_3(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3$$

$$f_4(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$$

$$f_5(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5$$

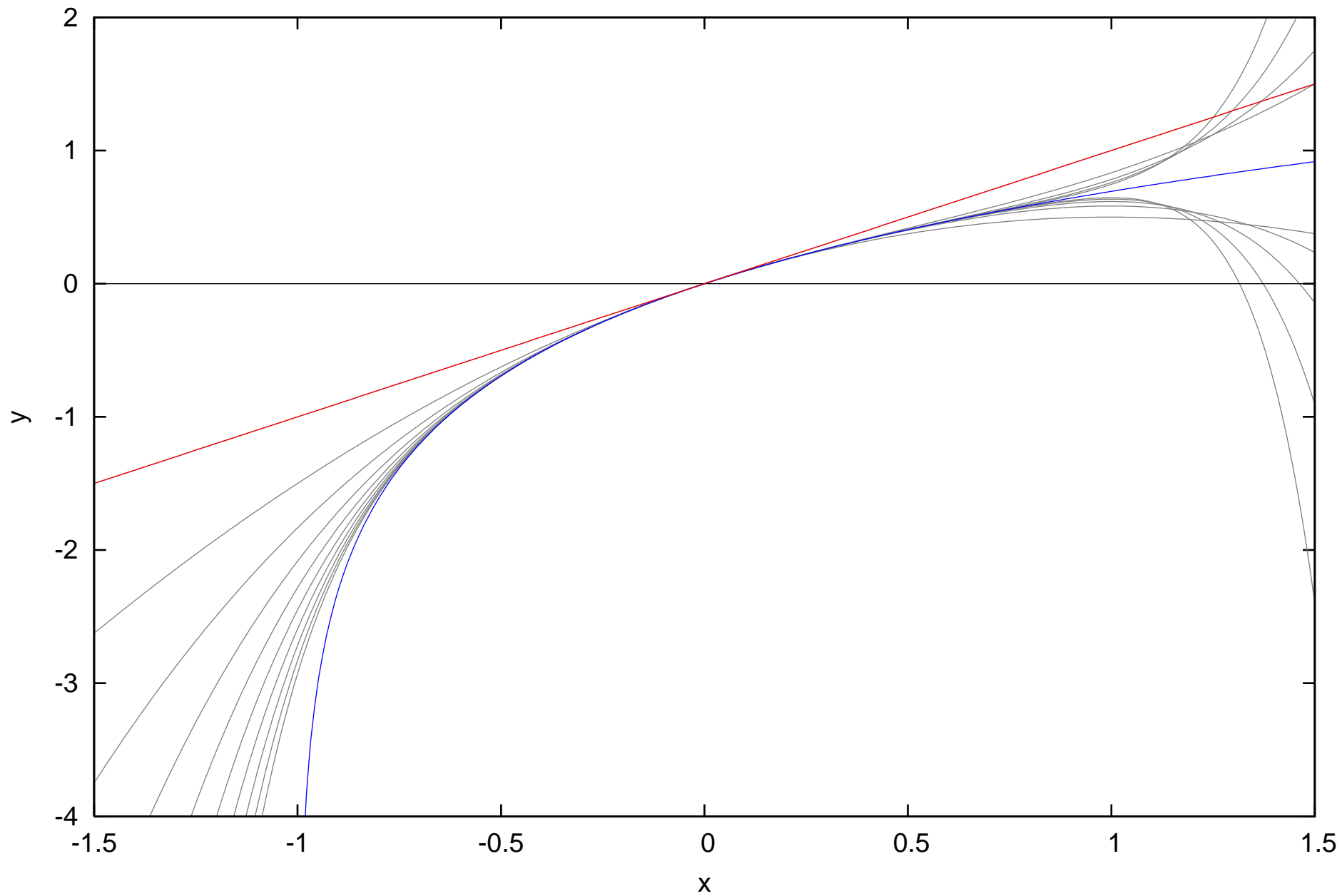
$$f_6(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \frac{1}{6}x^6$$

$$f_7(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \frac{1}{6}x^6 + \frac{1}{7}x^7$$

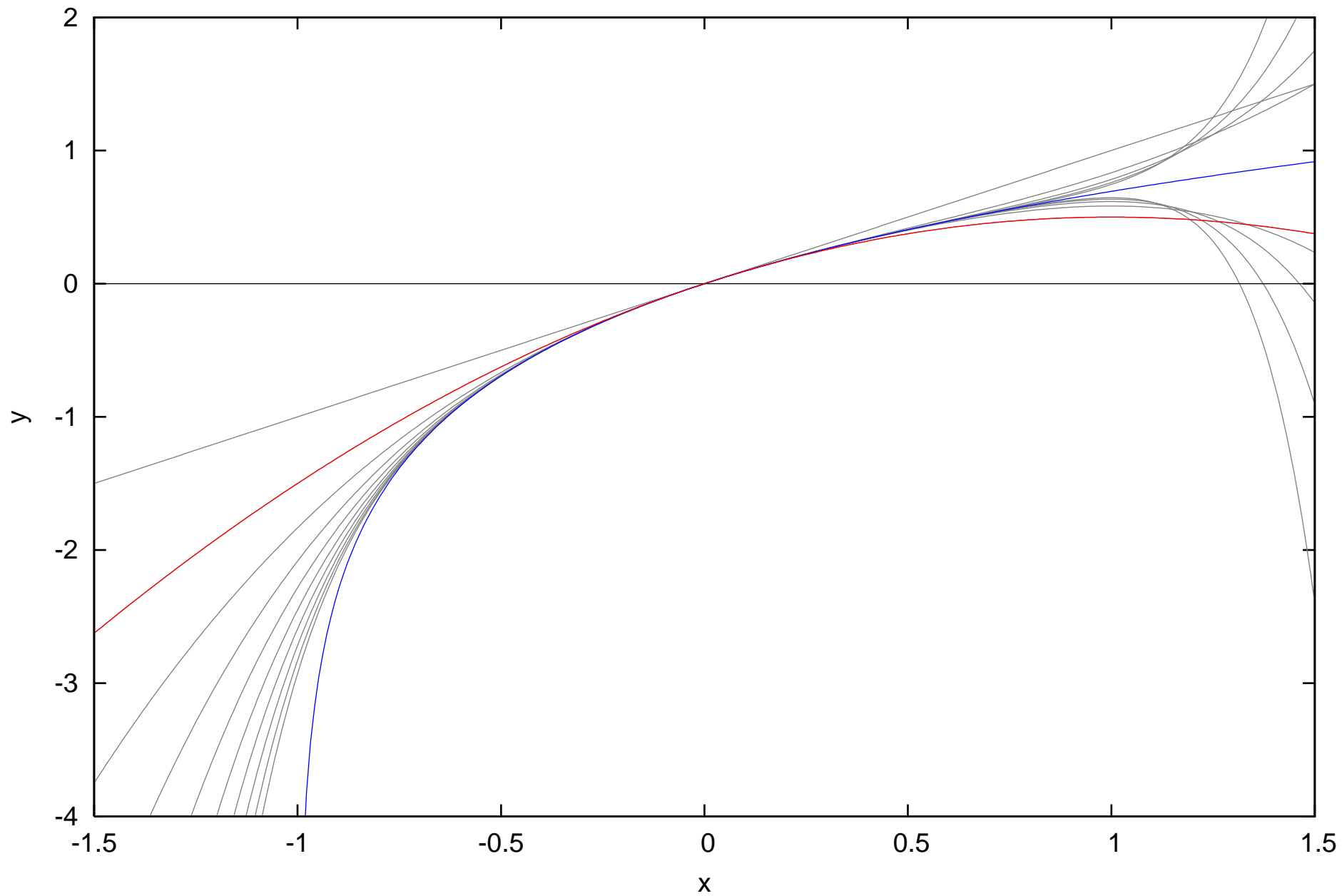
...

【注】 $\log(1+x)$ のテーラー展開の収束半径は 1 である。即ち、 $|x| < 1$ で $f_n(x) \rightarrow f(x)$ ($n \rightarrow \infty$) となるが、 $|x| > 1$ では $f_n(x)$ は $n \rightarrow \infty$ で発散する。

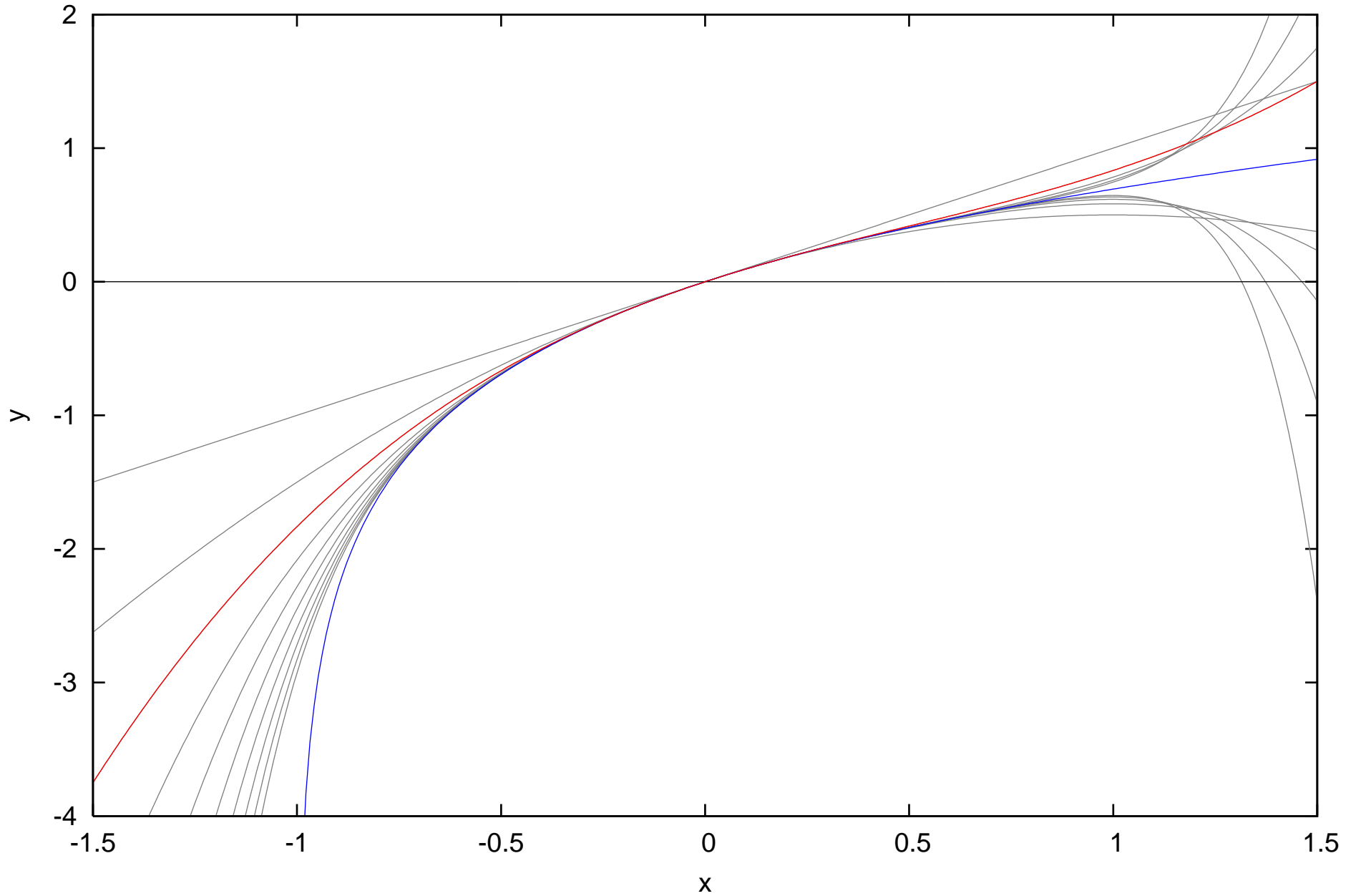
blue curve: $f(x)=\log(1+x)$, red curve: $f_1(x)=x$, gray curves: $f_2(x), f_3(x), \dots, f_{10}(x)$



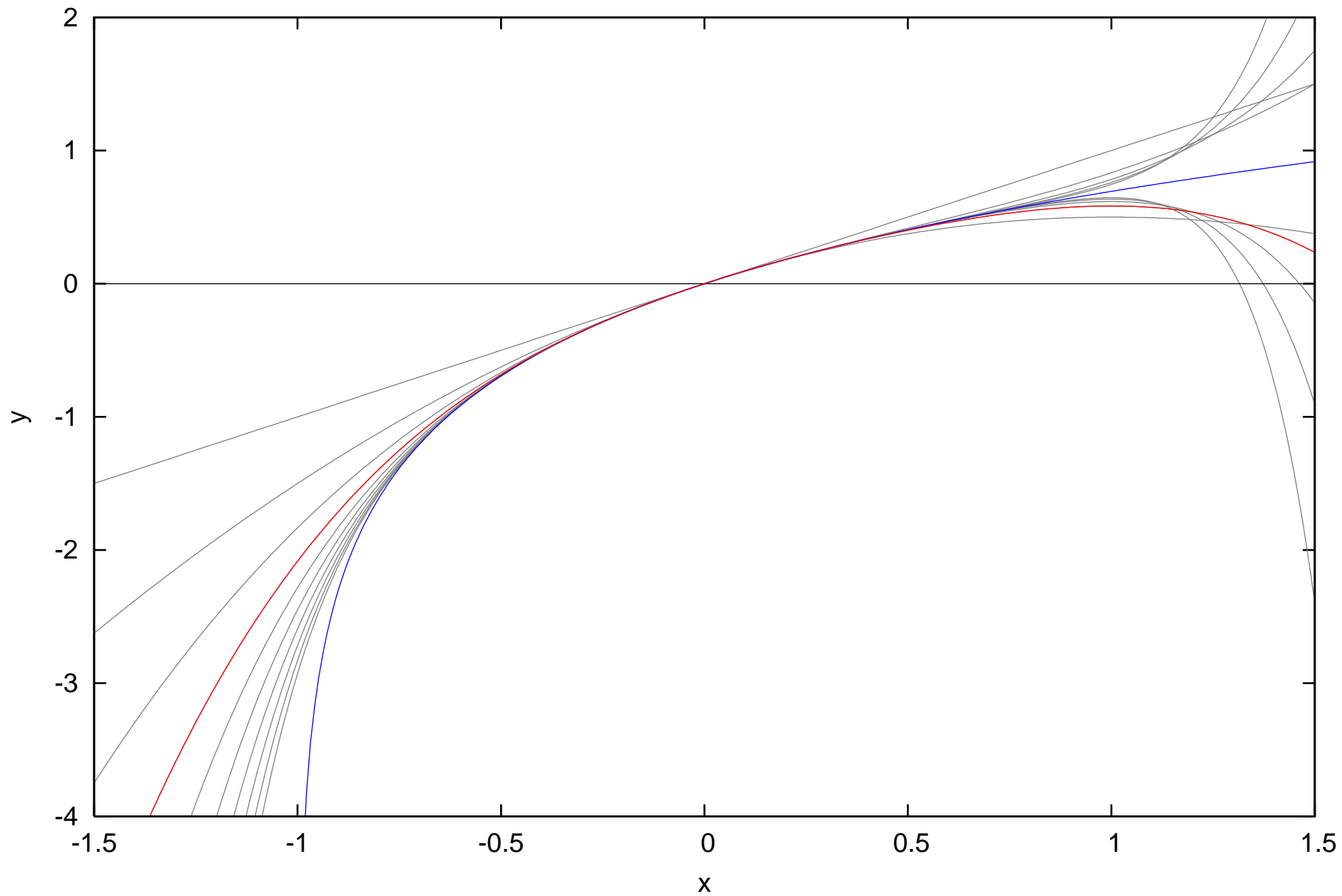
blue curve: $f(x)=\log(1+x)$, red curve: $f_2(x)=x-x^2/2$



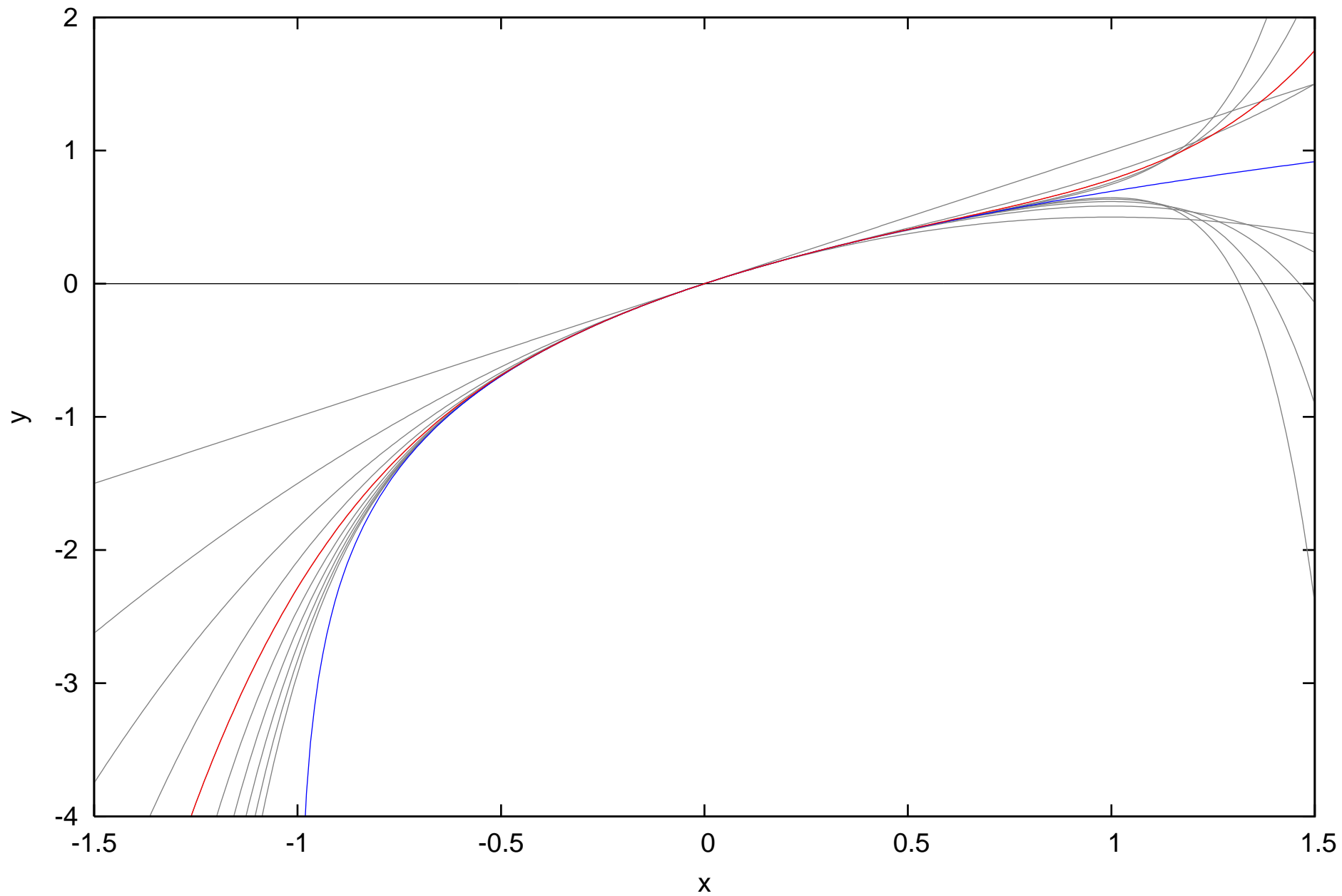
blue curve: $f(x)=\log(1+x)$, red curve: $f_3(x)=x-x^2/2+x^3/3$



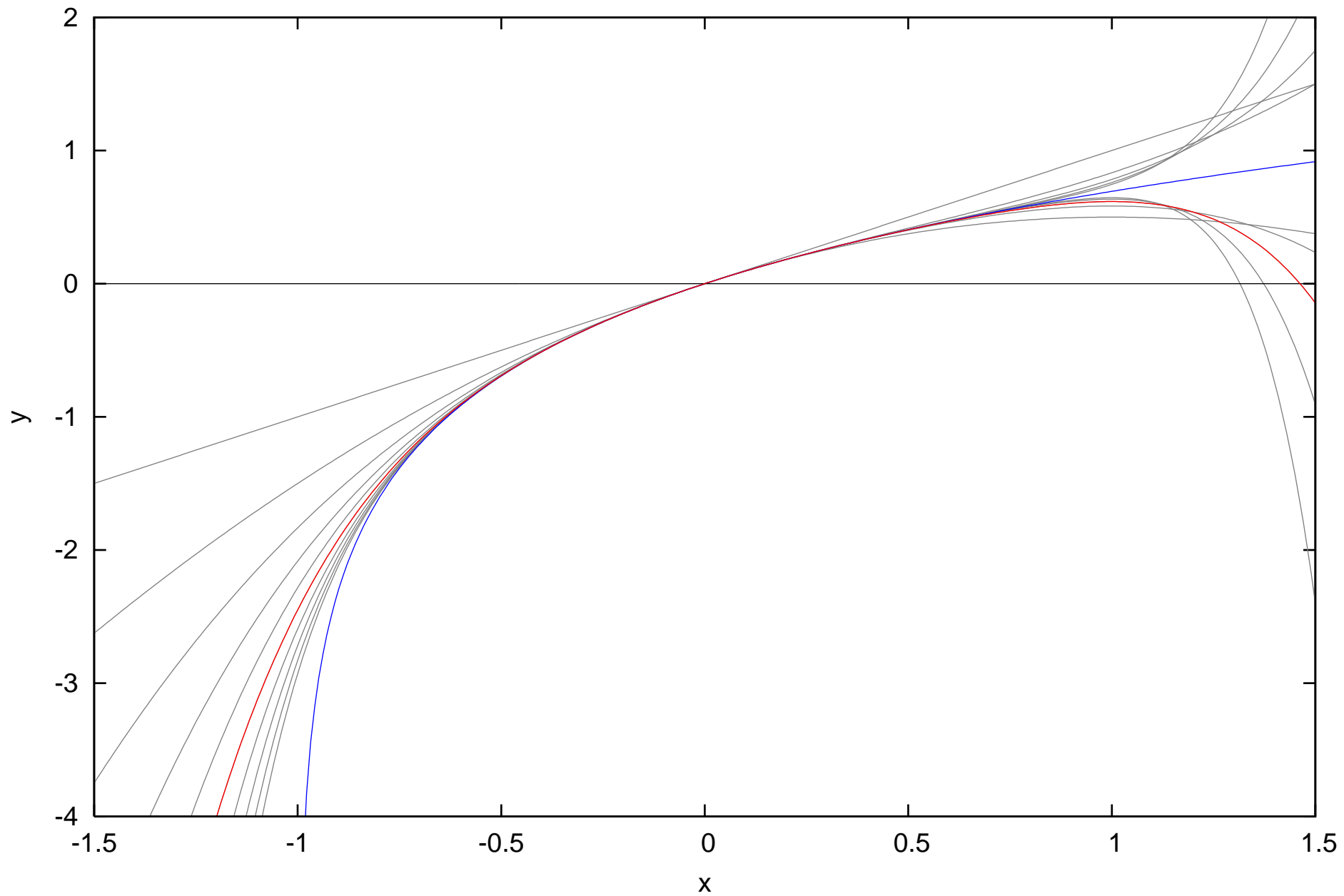
blue curve: $f(x)=\log(1+x)$, red curve: $f_4(x)=x-x^2/2+x^3/3-x^4/4$



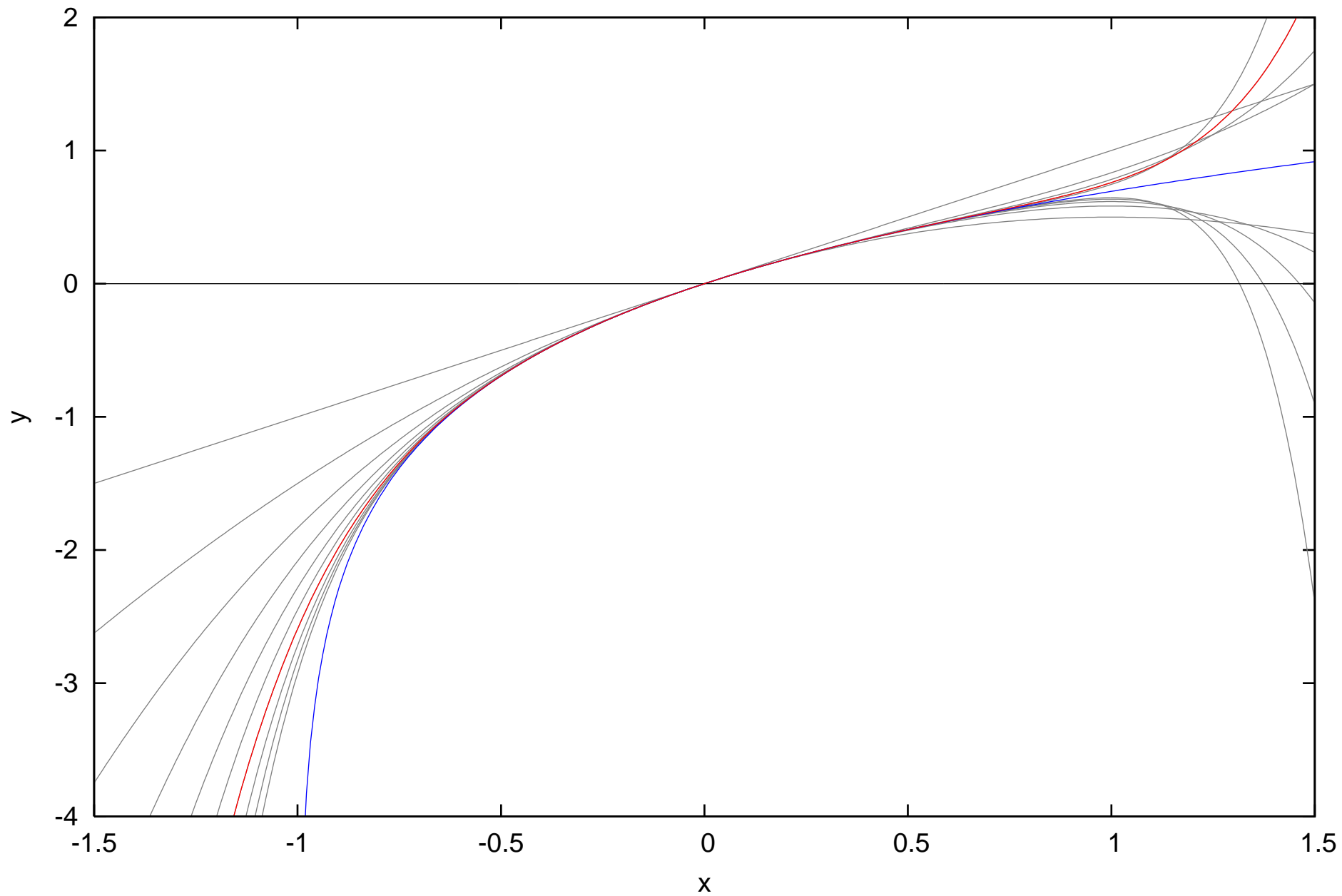
blue curve: $f(x)=\log(1+x)$, red curve: $f_5(x)$



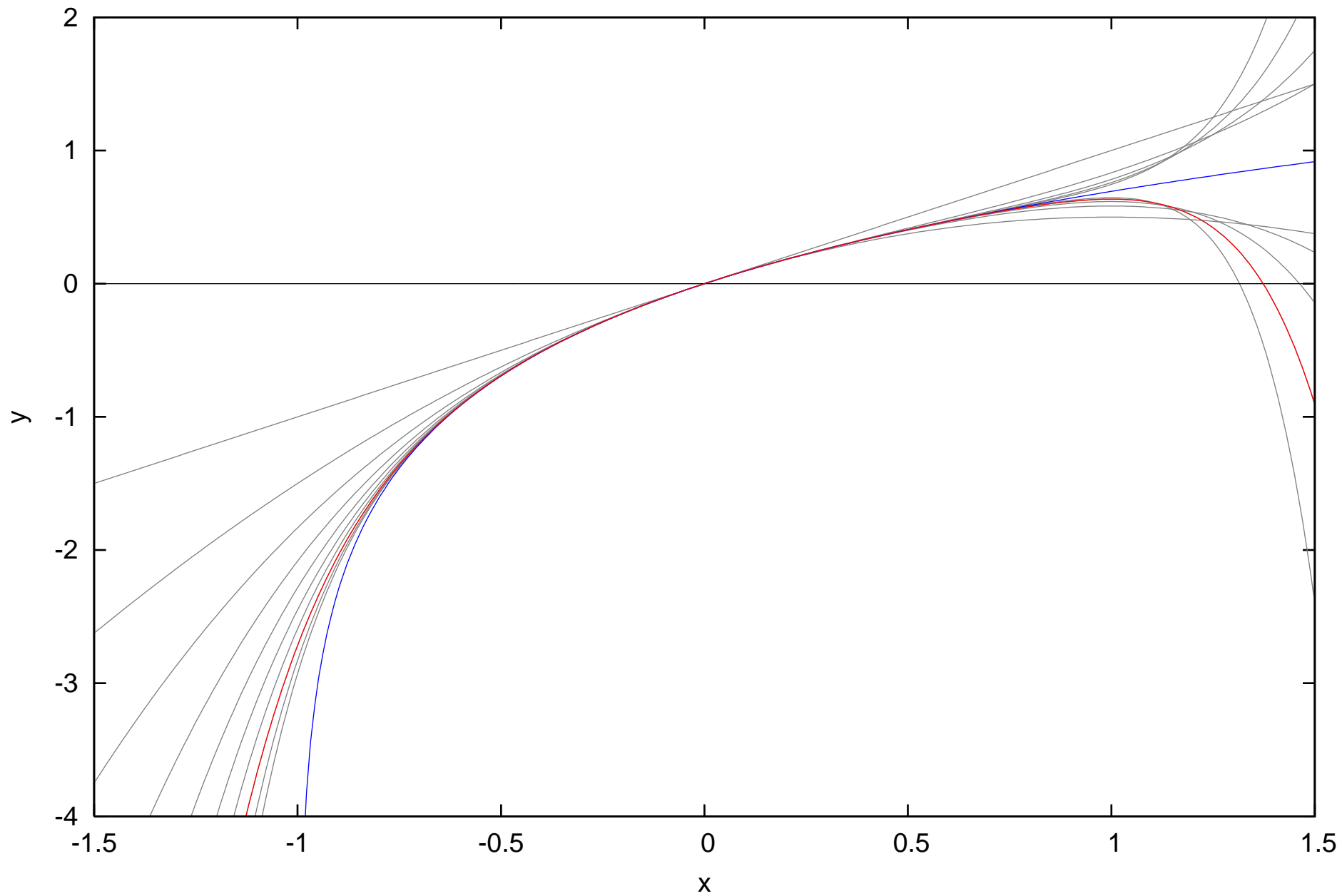
blue curve: $f(x)=\log(1+x)$, red curve: $f_6(x)$



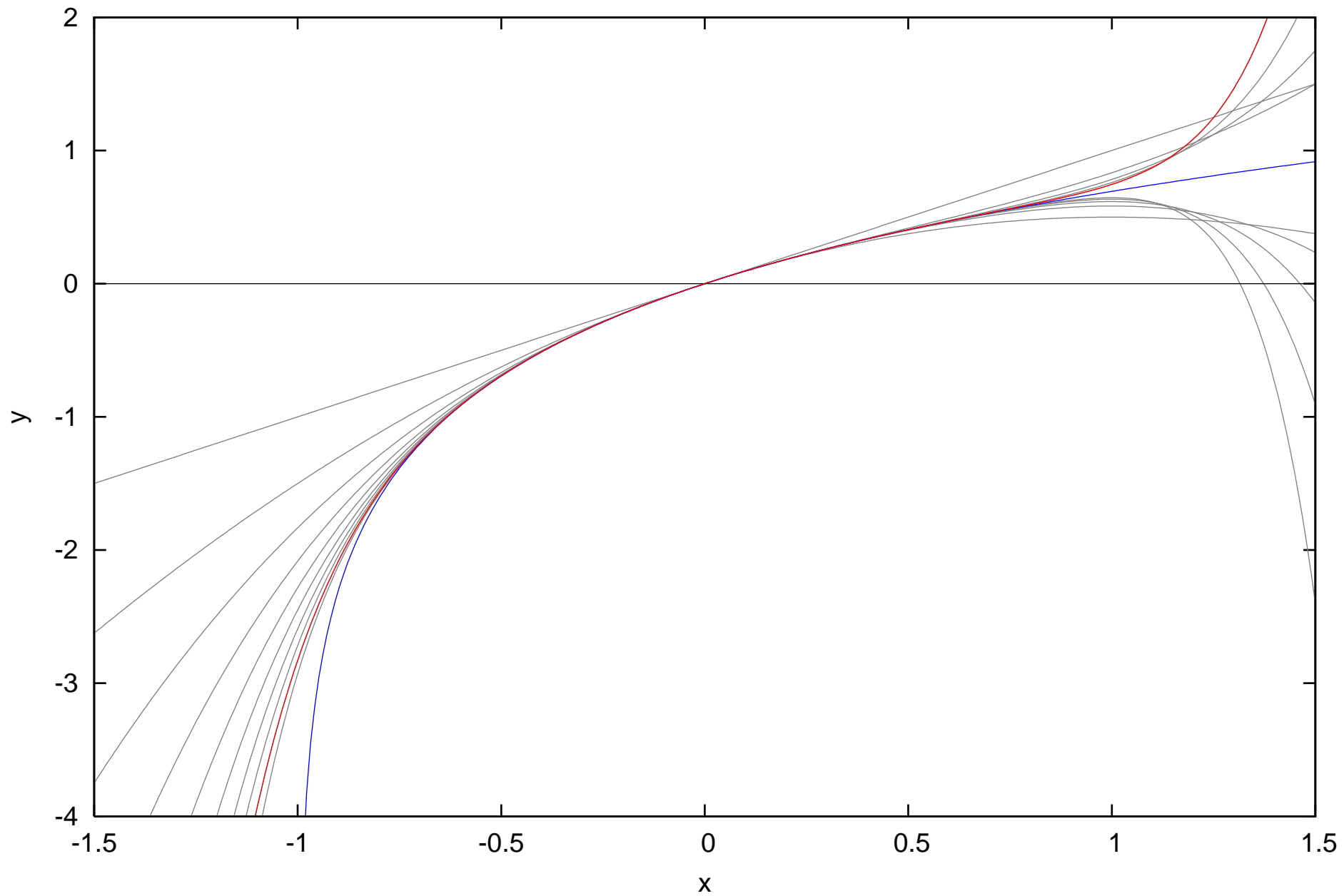
blue curve: $f(x)=\log(1+x)$, red curve: $f_7(x)$



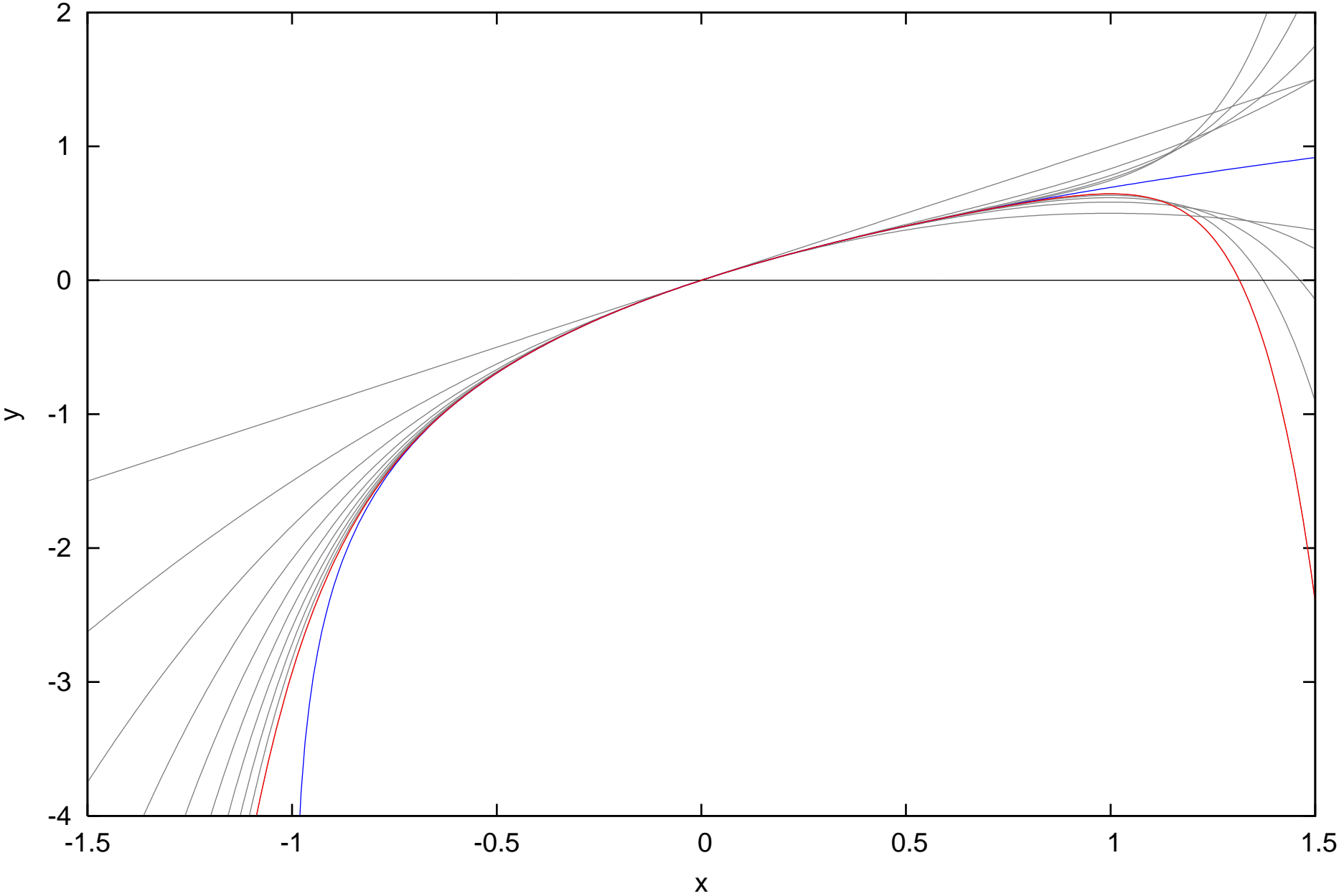
blue curve: $f(x)=\log(1+x)$, red curve: $f_8(x)$



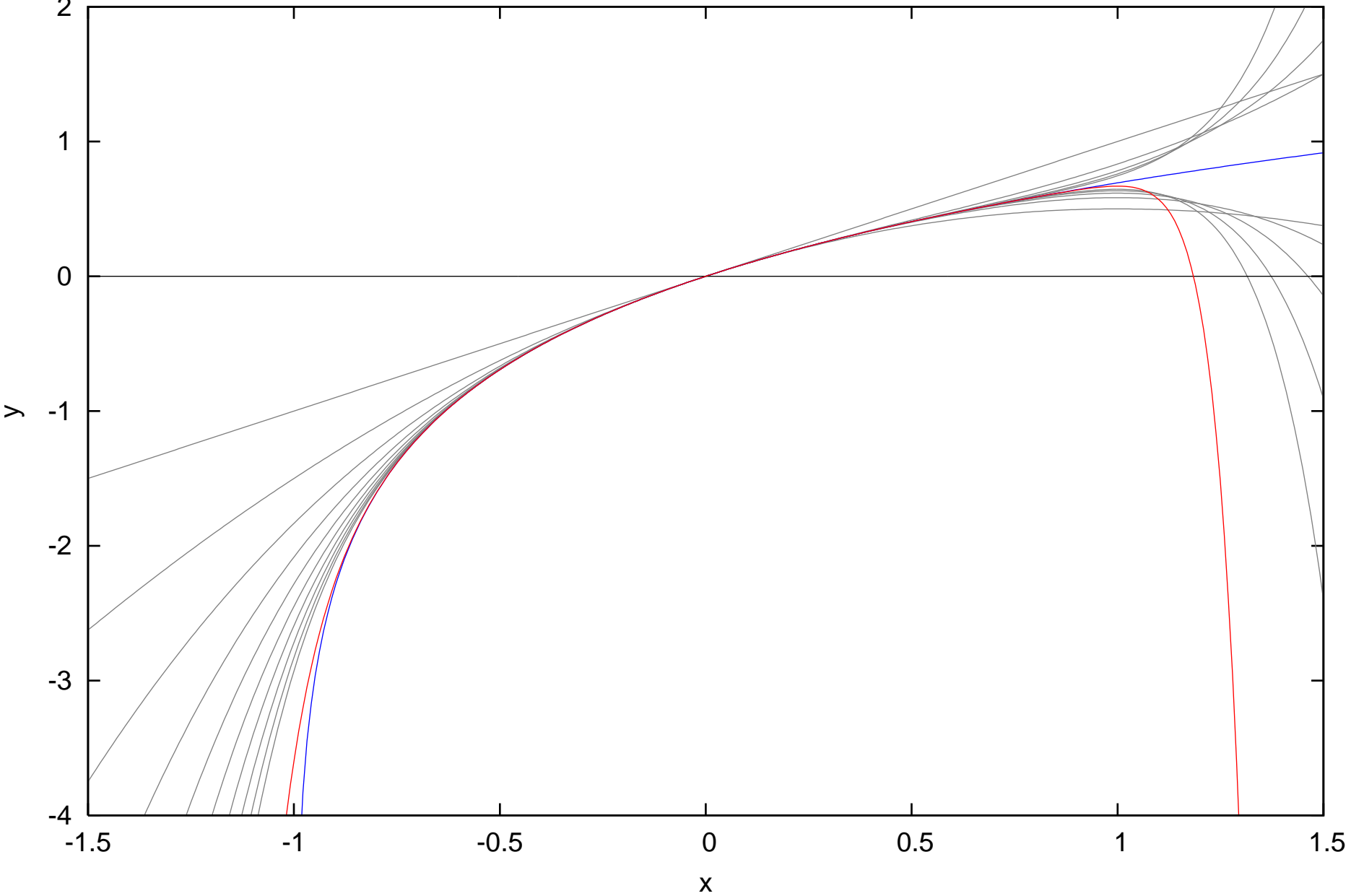
blue curve: $f(x)=\log(1+x)$, red curve: $f_9(x)$



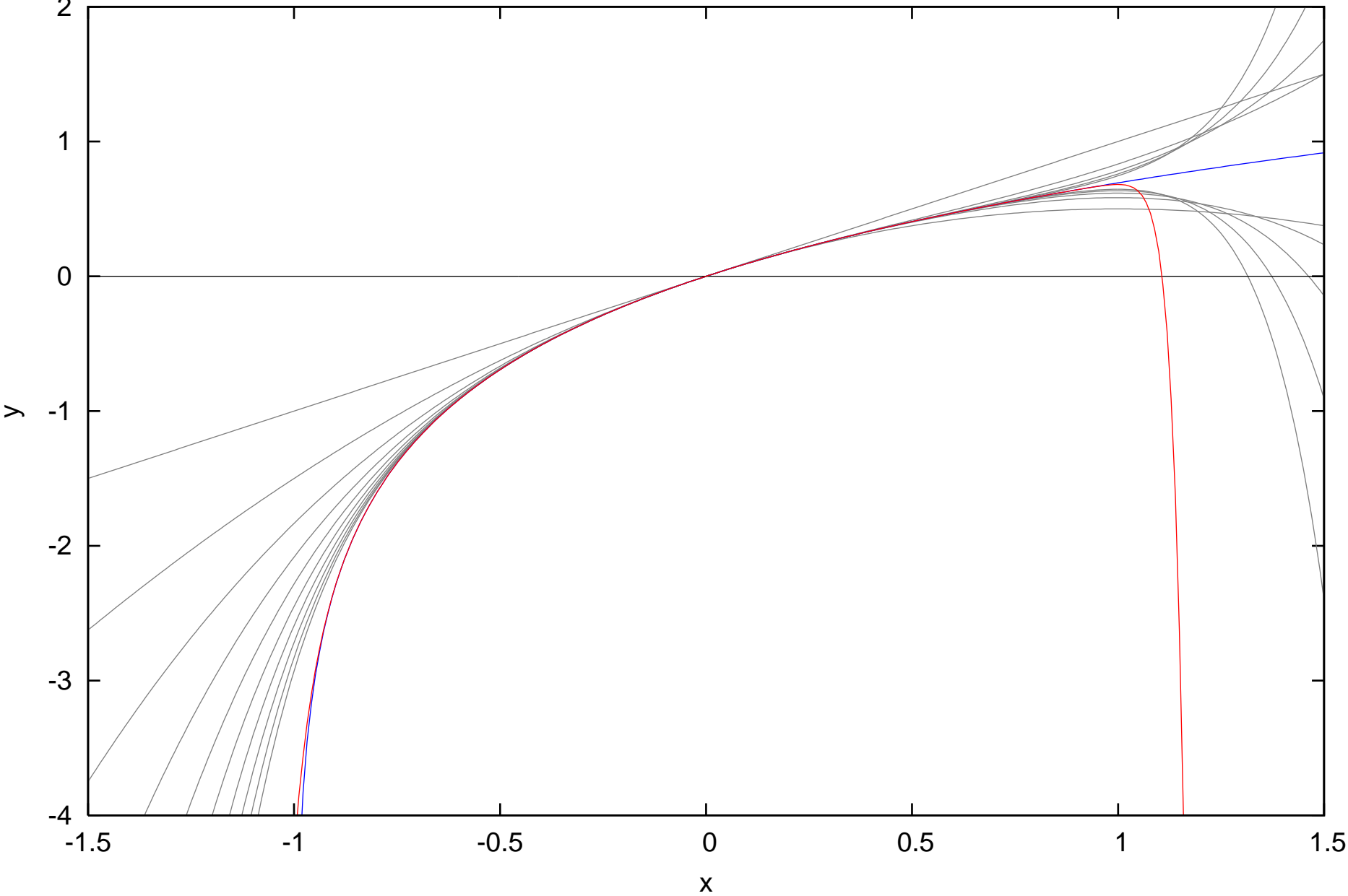
blue curve: $f(x)=\log(1+x)$, red curve: $f_{10}(x)$



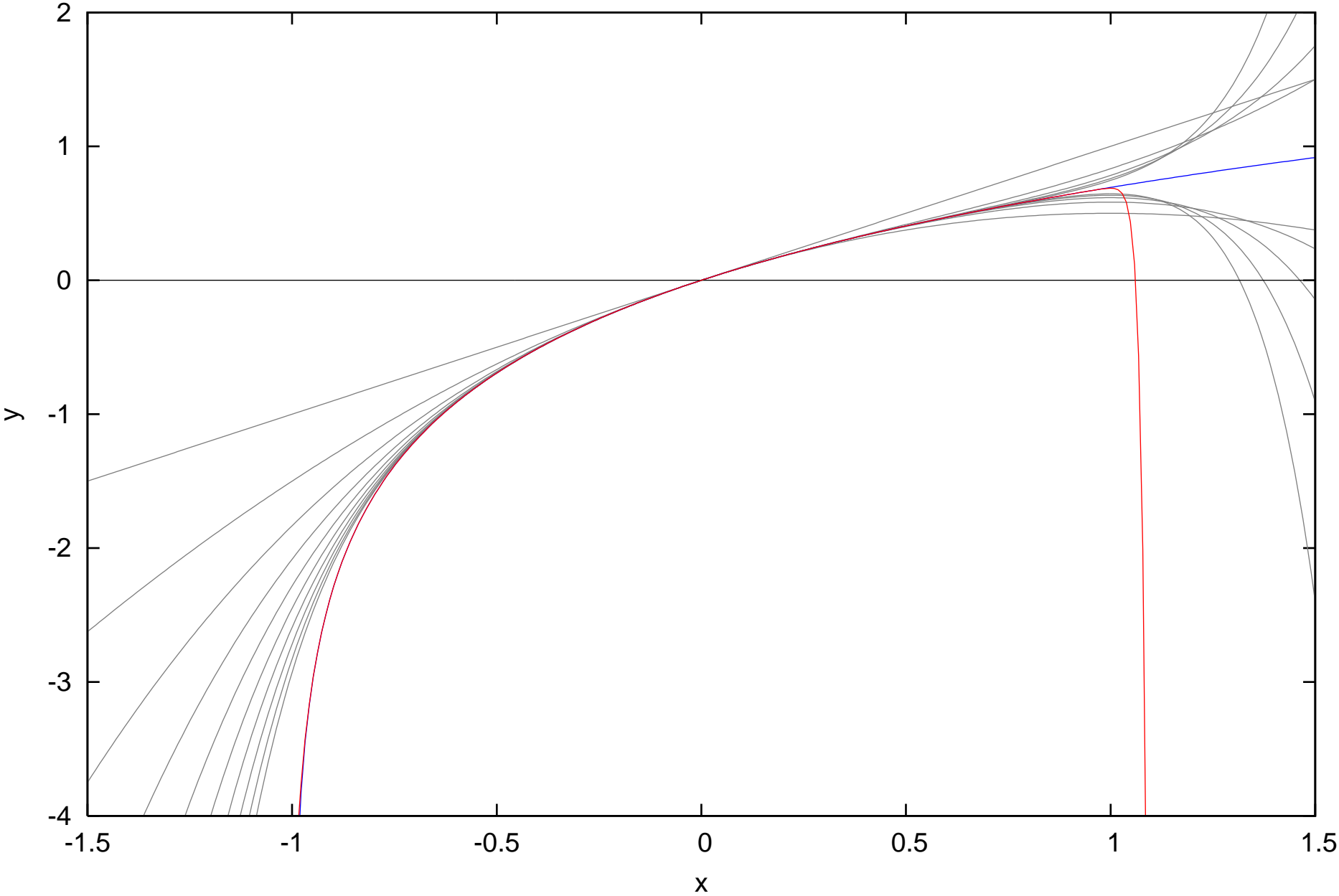
blue curve: $f(x)=\log(1+x)$, red curve: $f_{20}(x)$



blue curve: $f(x)=\log(1+x)$, red curve: $f_{40}(x)$



blue curve: $f(x)=\log(1+x)$, red curve: $f_{80}(x)$



blue curve: $f(x)=\log(1+x)$, red curve: $f_{81}(x)$

