テーラー展開のグラフ　：　$y = f(x) = \cos\, x$ の場合

$f(x) = \cos\, x$

$f_n(x) = \displaystyle\sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k$　：　$f(x)$ のテーラー展開（級数）の x の べき が 0 乗から n 乗までの項の和。

具体的な形は：

$f_0(x) = 1$

$f_2(x) = 1 - \frac{1}{2}x^2$

$f_4(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$

$f_6(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6$

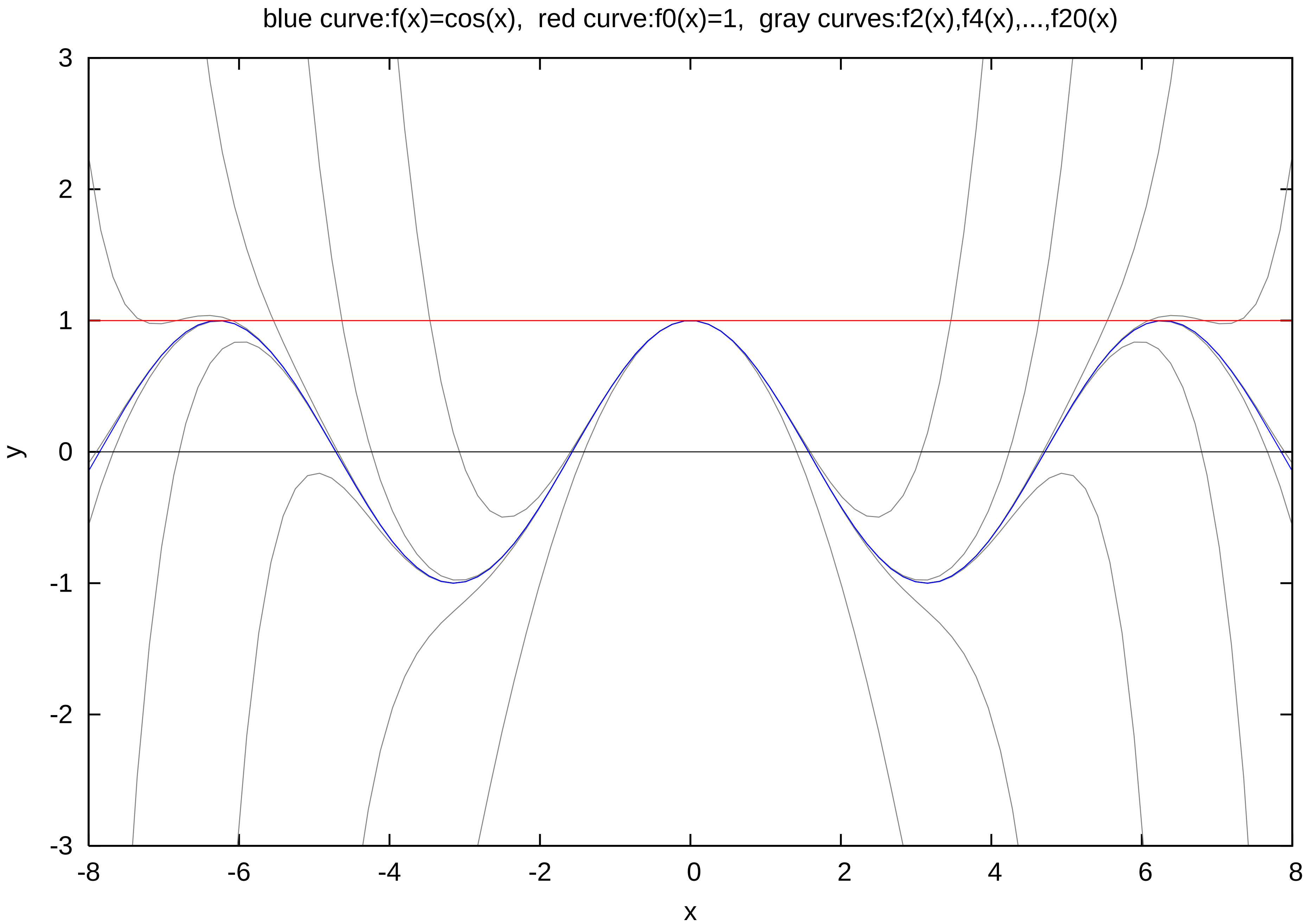
$f_8(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \frac{1}{40320}x^8$

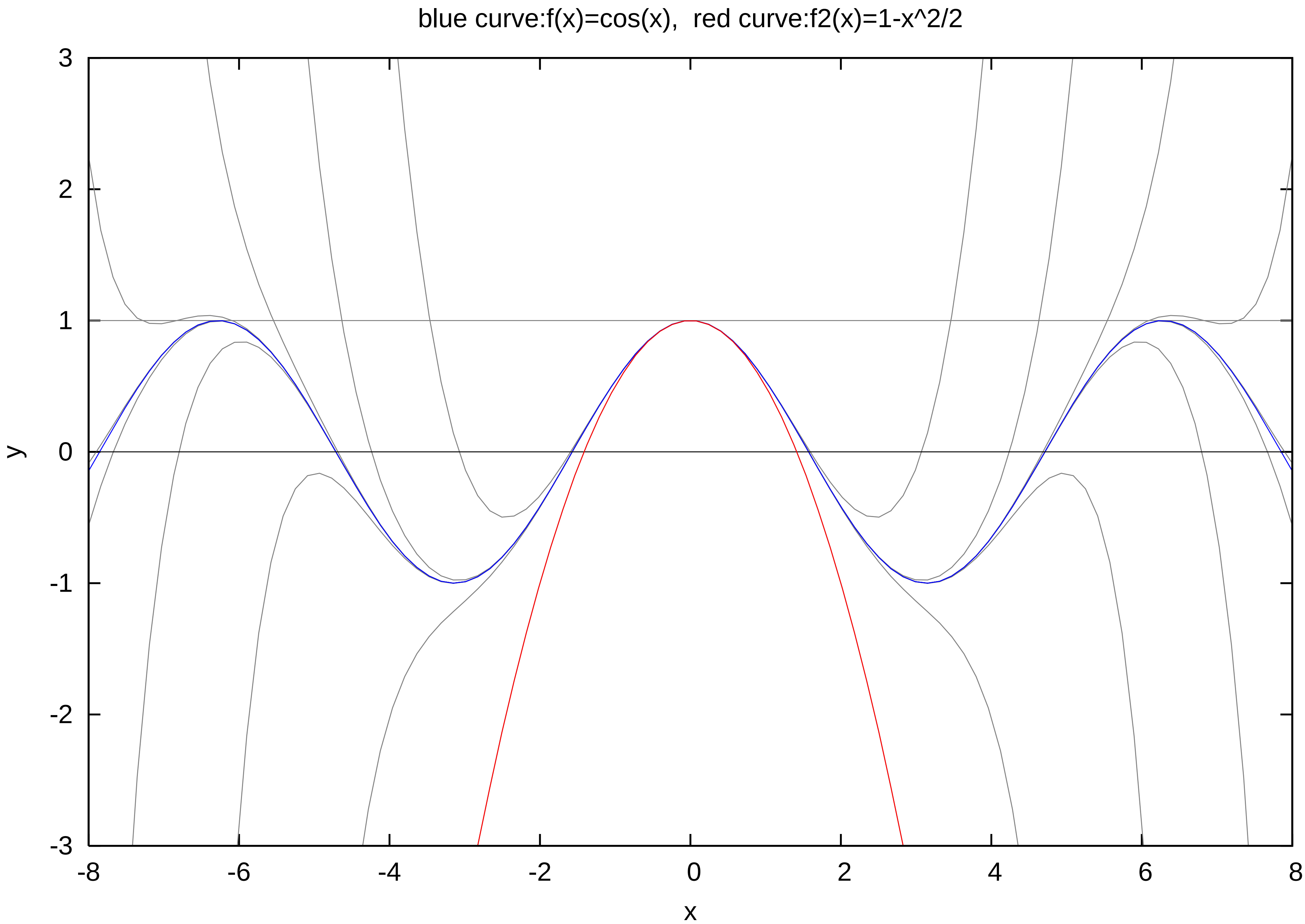
$f_{10}(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \frac{1}{40320}x^8 - \frac{1}{3628800}x^{10}$

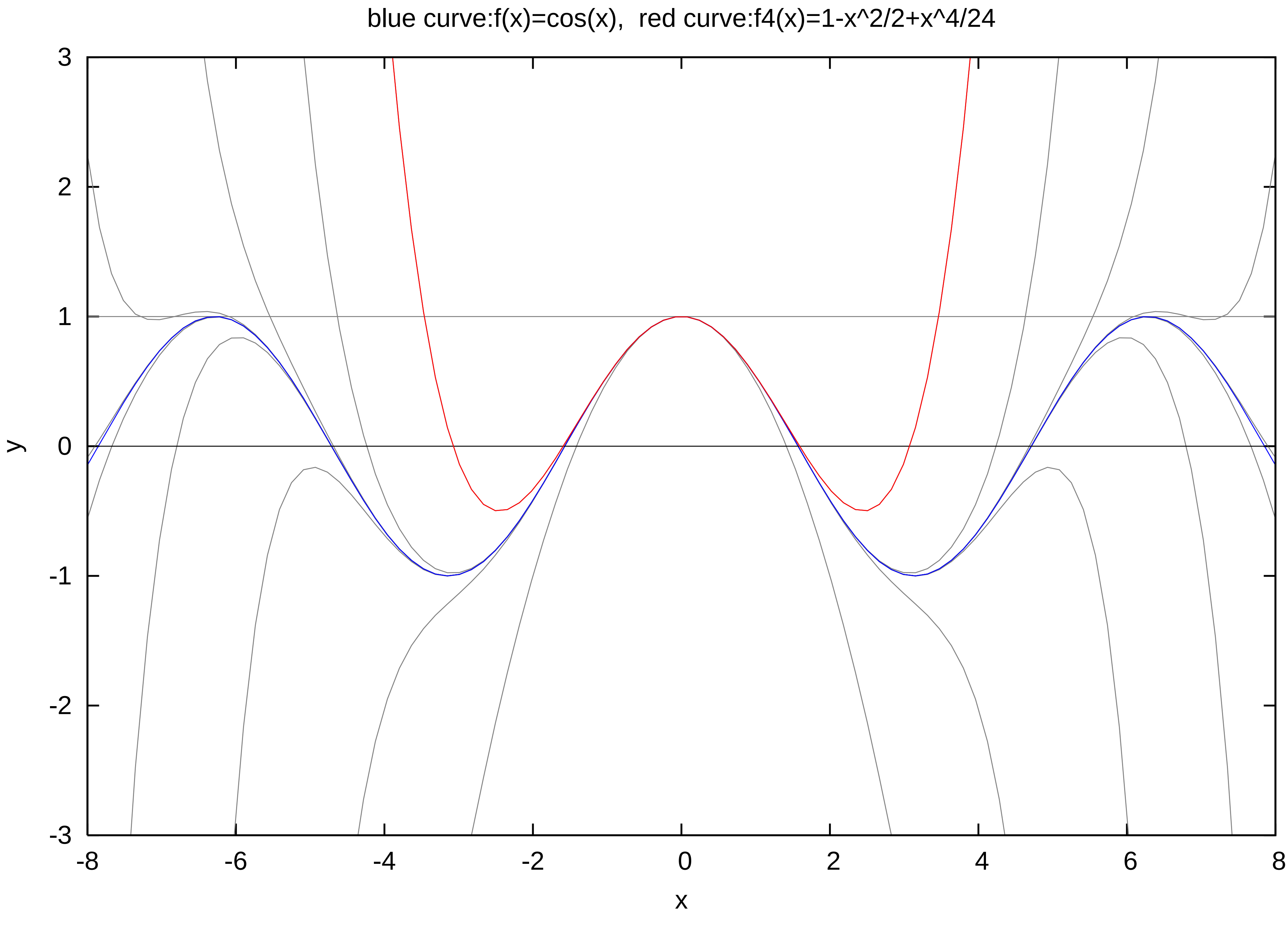
$f_{12}(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \frac{1}{40320}x^8 - \frac{1}{3628800}x^{10} + \frac{1}{479001600}x^{12}$

$\cdots$

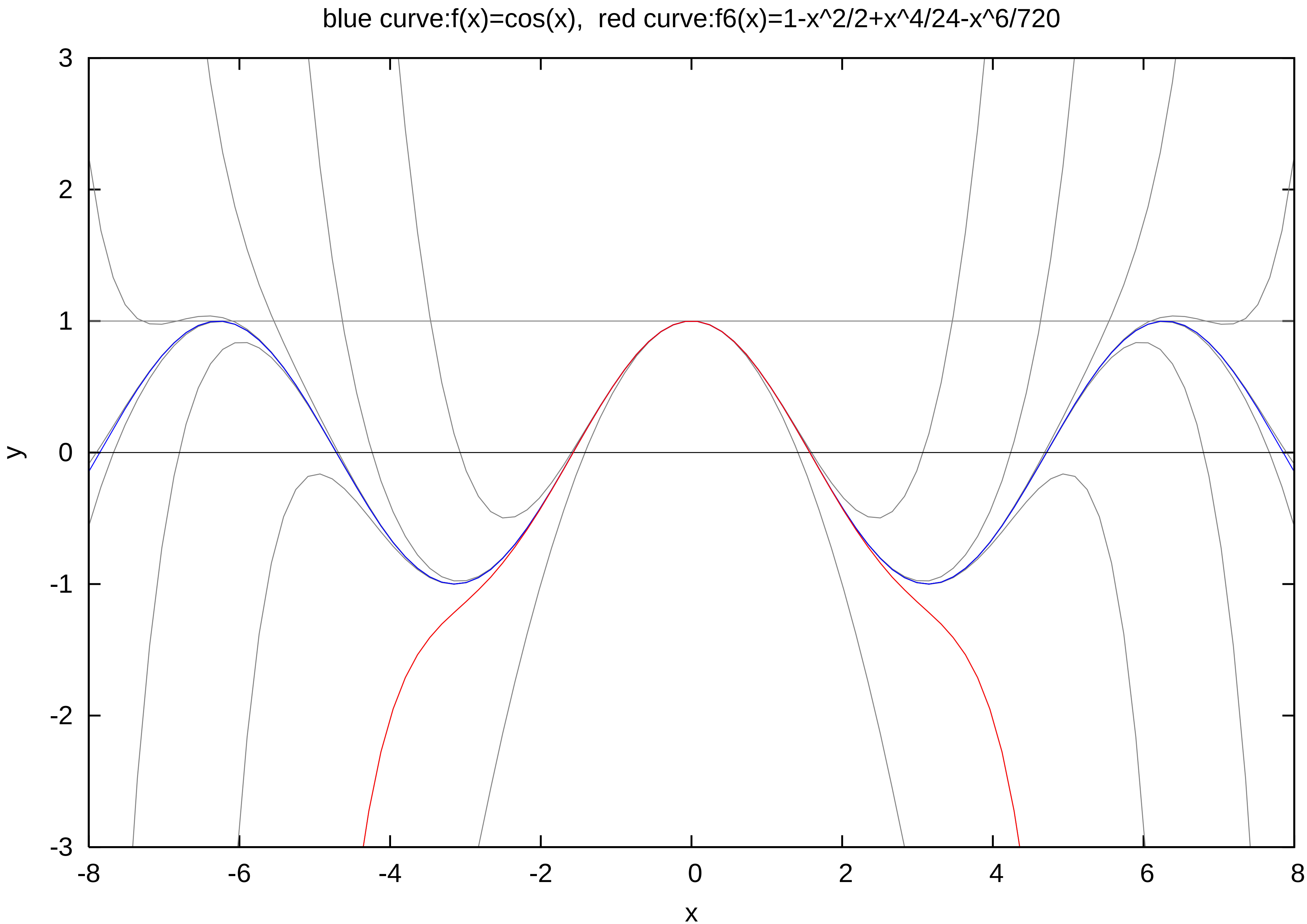
【注】$\cos\, x$ のように偶関数のテーラー展開では、x の奇数べきの項の係数はゼロとなる。したがって、$f_1(x) = f_0(x),\ f_3(x) = f_2(x),\ f_5(x) = f_4(x),\ \cdots$ である。


blue curve:f(x)=cos(x), red curve:f0(x)=1, gray curves:f2(x),f4(x),...,f20(x)
y
x
3
2
1
0
-1
-2
-3
-8
-6
-4
-2
0
2
4
6
8

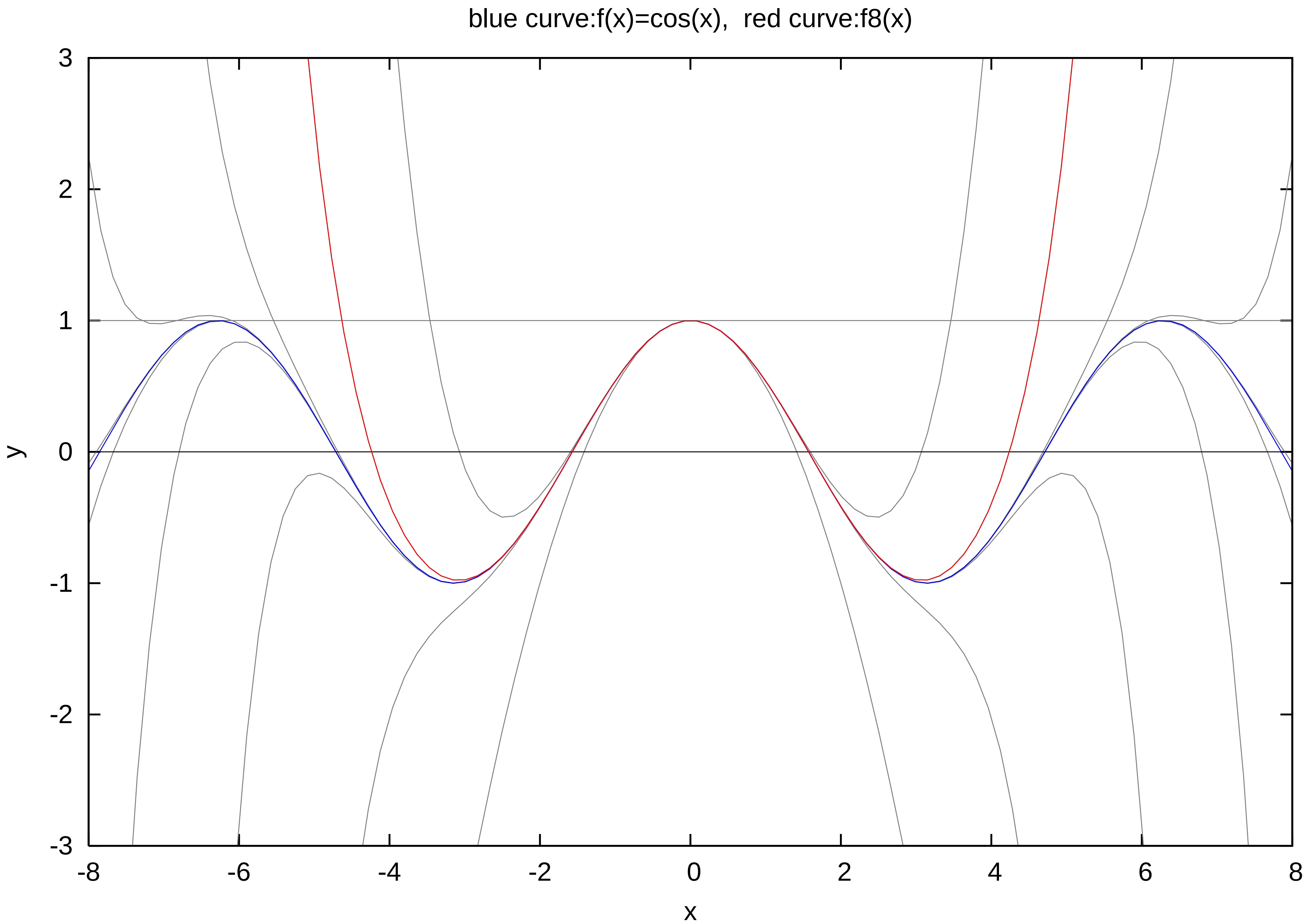

blue curve:f(x)=cos(x), red curve:f2(x)=1-x^2/2
3
2
1
0
-1
-2
-3
y
-8
-6
-4
-2
0
2
4
6
8
x



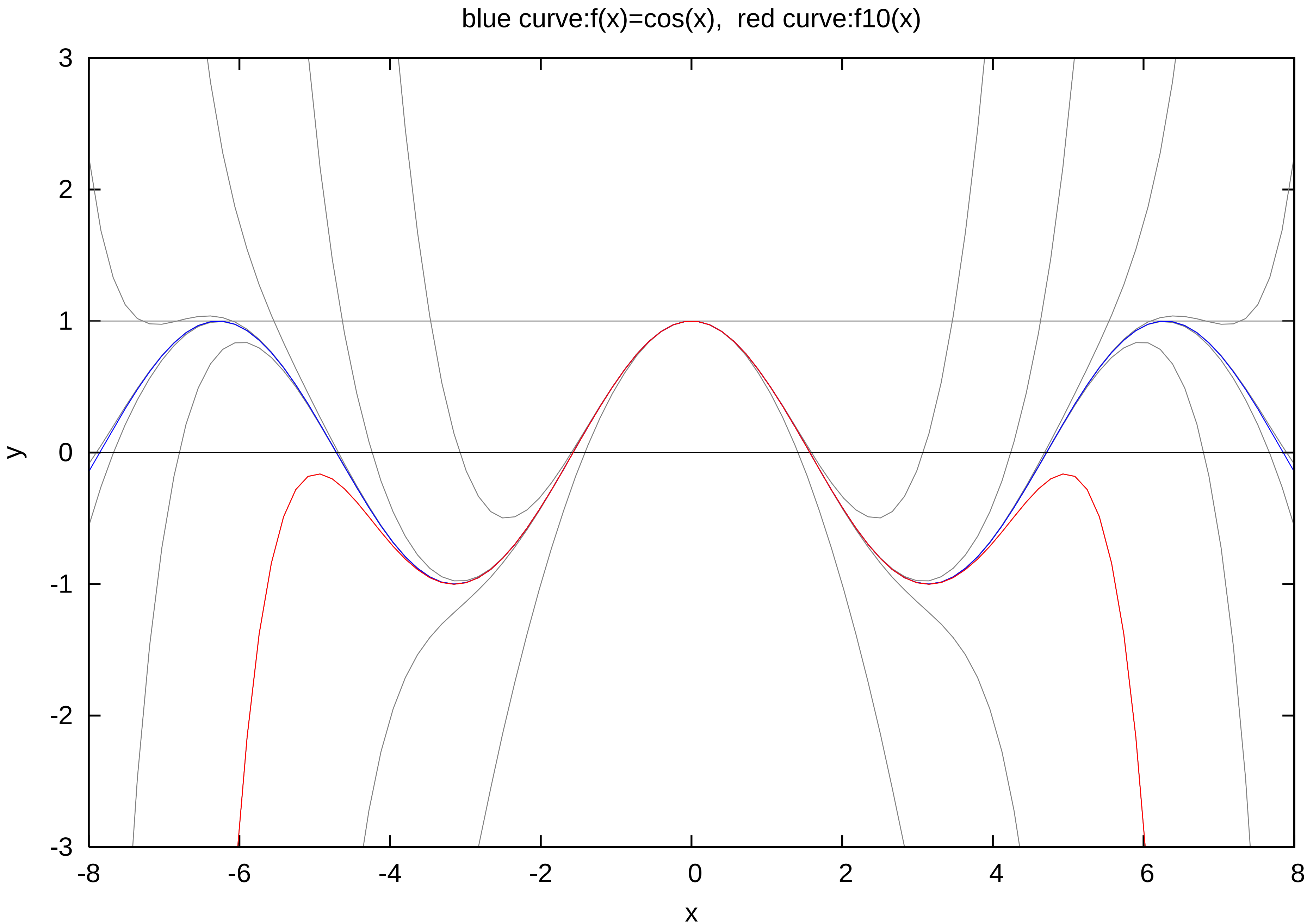
blue curve:f(x)=cos(x), red curve:f4(x)=1-x^2/2+x^4/24
3
2
1
0
-1
-2
-3
y
-8
-6
-4
-2
0
2
4
6
8
x

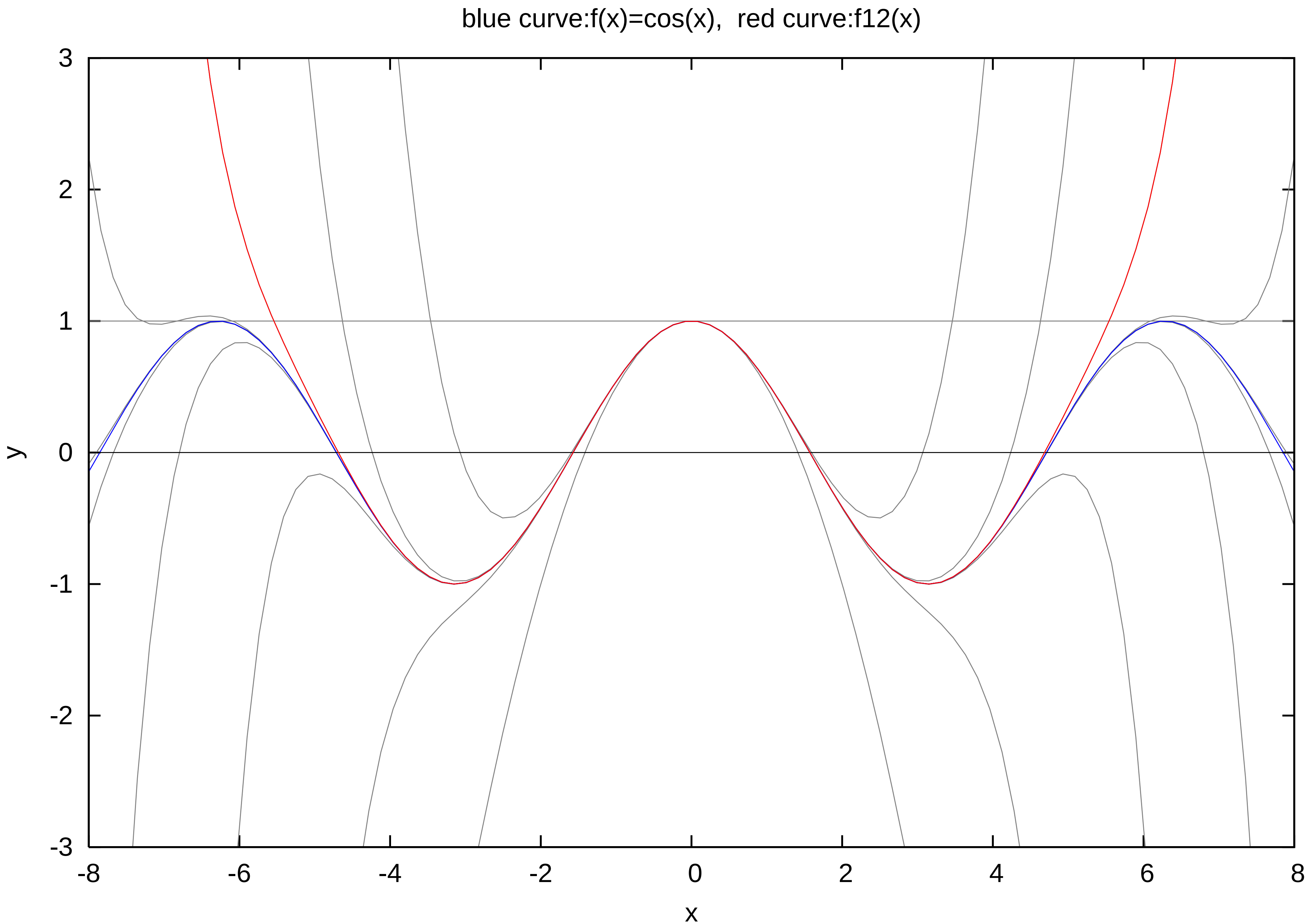


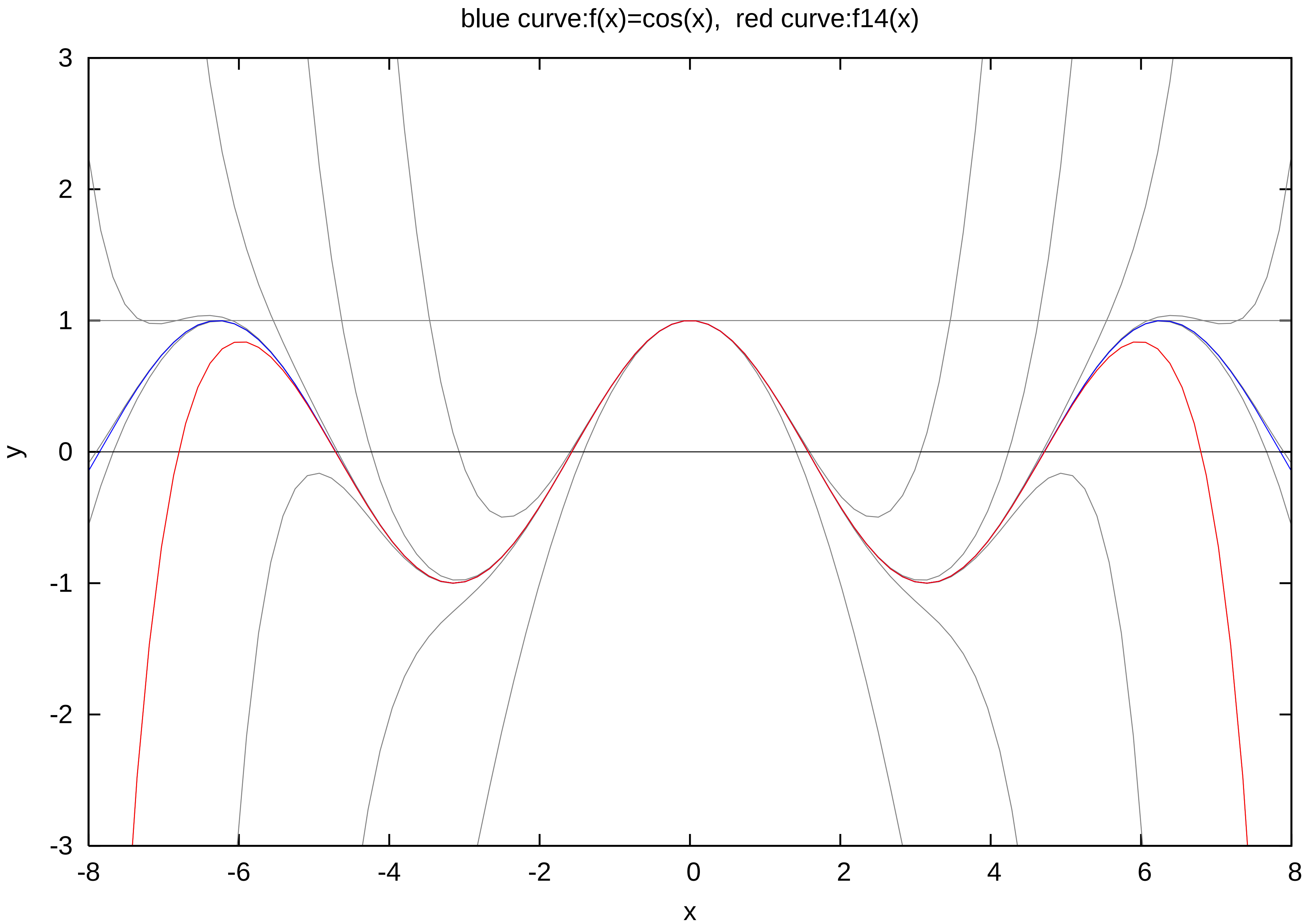
blue curve:f(x)=cos(x), red curve:f6(x)=1-x^2/2+x^4/24-x^6/720
3
2
1
0
-1
-2
-3
y
-8
-6
-4
-2
0
2
4
6
8
x

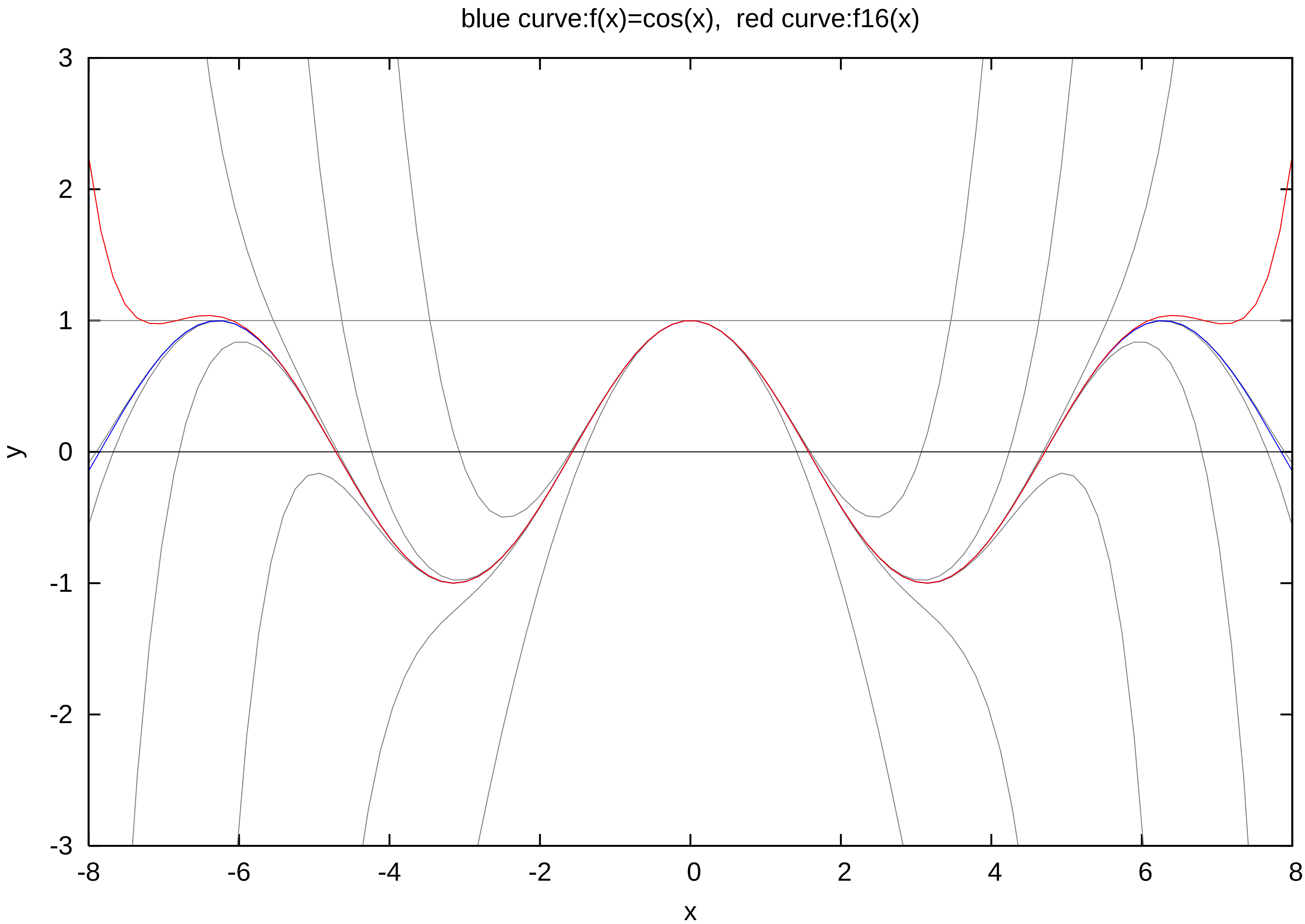


blue curve:f(x)=cos(x), red curve:f8(x)
3
2
1
0
-1
-2
-3
y
-8
-6
-4
-2
0
2
4
6
8
x

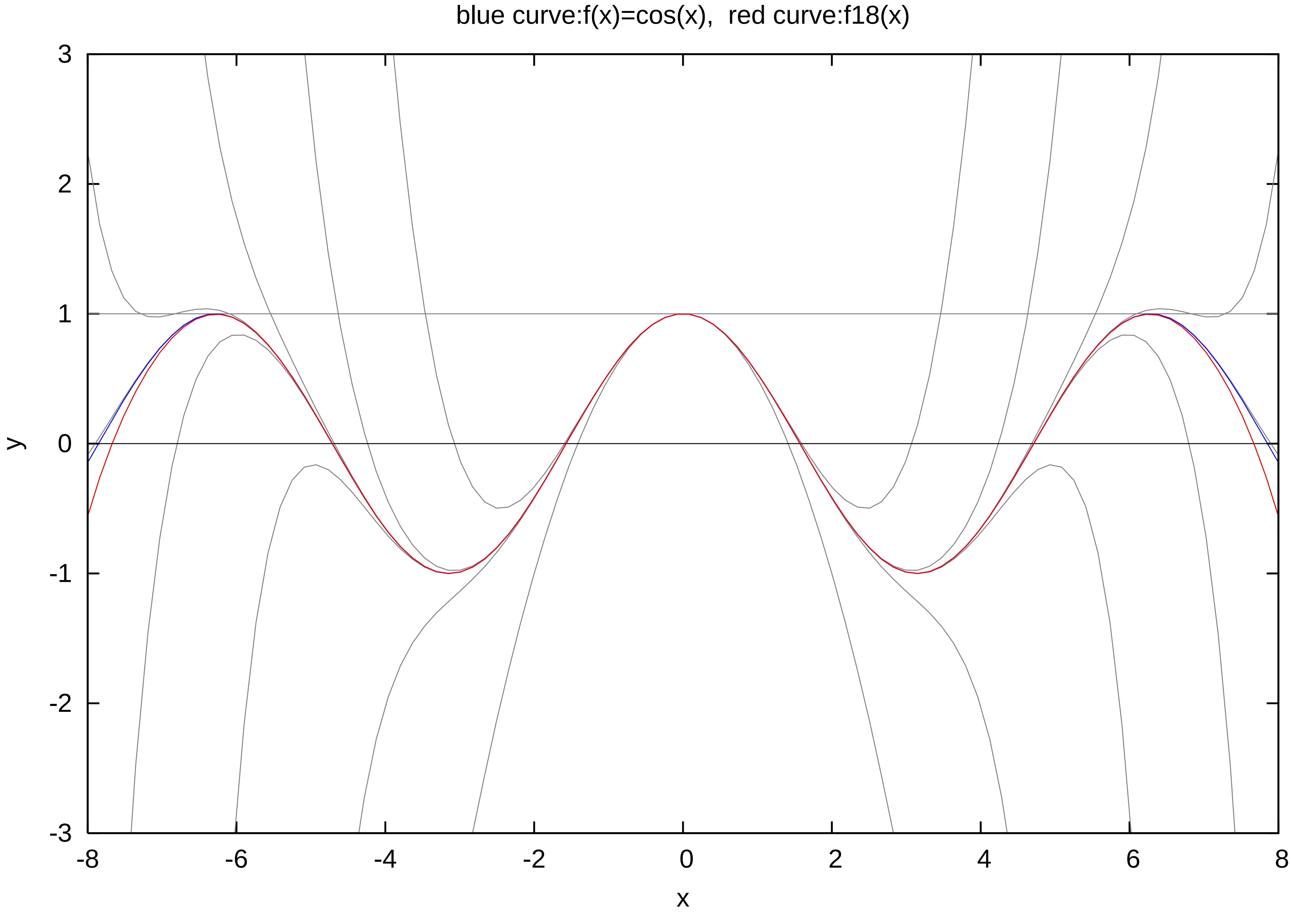

blue curve:f(x)=cos(x), red curve:f10(x)
3
2
1
0
-1
-2
-3
y
-8
-6
-4
-2
0
2
4
6
8
x

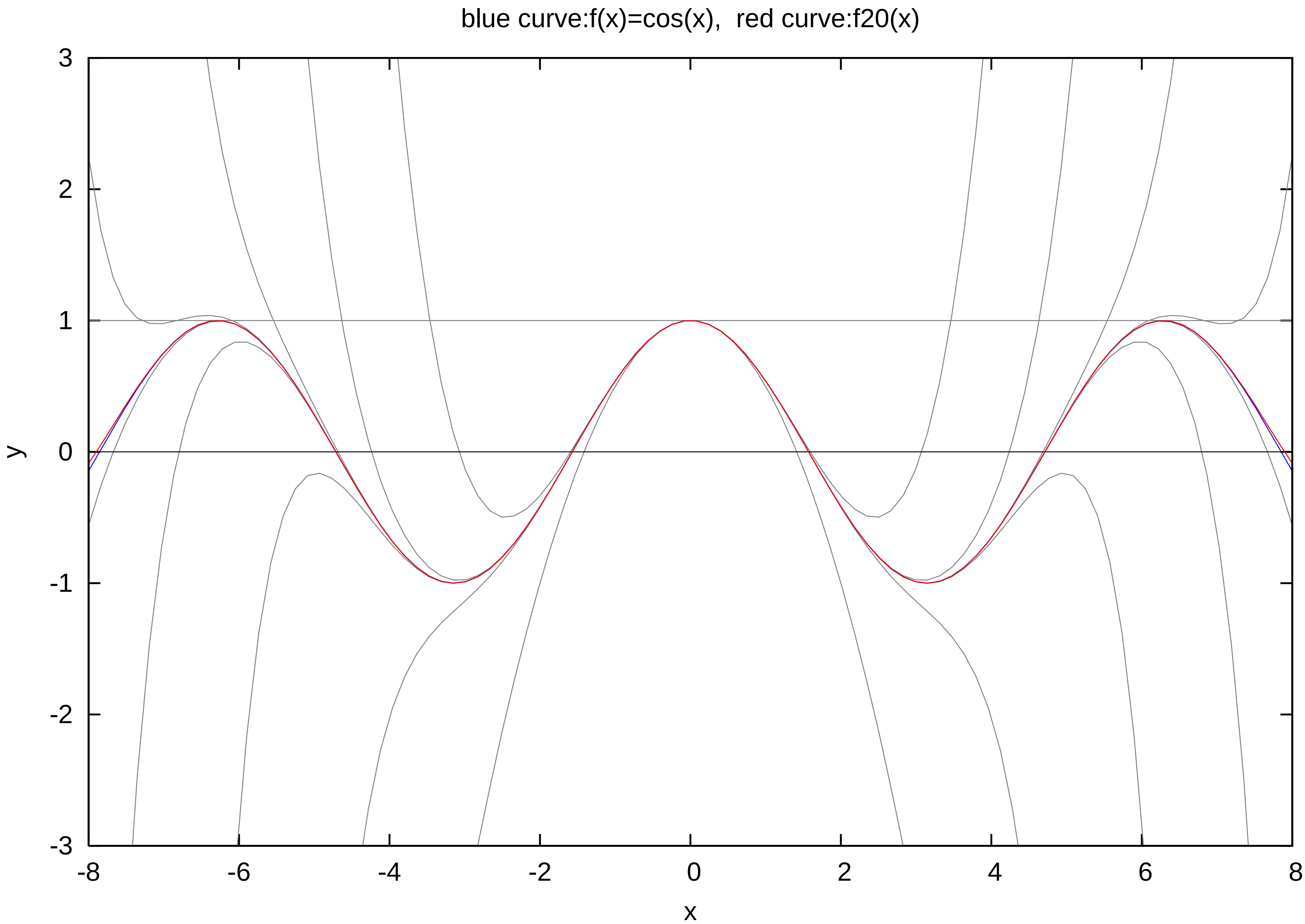

blue curve:f(x)=cos(x), red curve:f12(x)
y
x
3
2
1
0
-1
-2
-3
-8
-6
-4
-2
0
2
4
6
8


blue curve:f(x)=cos(x), red curve:f14(x)
3
2
1
0
-1
-2
-3
y
-8
-6
-4
-2
0
2
4
6
8
x



blue curve:f(x)=cos(x), red curve:f16(x)
y
x
3
2
1
0
-1
-2
-3
-8
-6
-4
-2
0
2
4
6
8


blue curve:f(x)=cos(x), red curve:f18(x)
3
2
1
0
-1
-2
-3
y
-8
-6
-4
-2
0
2
4
6
8
x


blue curve:f(x)=cos(x), red curve:f20(x)
y
x
3
2
1
0
-1
-2
-3
-8
-6
-4
-2
0
2
4
6
8