

テーラー展開のグラフ :  $y = f(x) = (1 + x)^{-1/3}$  の場合

$$f(x) = (1 + x)^{-1/3}$$

$$f_n(x) = \sum_{k=0}^n \frac{f^{(k)}(0)}{k!} x^k \quad : \quad f(x) \text{ のテーラー展開 ( 級数 ) の } x \text{ の べき が } 0 \text{ 乗から } n \text{ 乗までの項の和。}$$

具体的な形は :

$$f_1(x) = 1 - \frac{1}{3}x$$

$$f_2(x) = 1 - \frac{1}{3}x + \frac{2}{9}x^2$$

$$f_3(x) = 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3$$

$$f_4(x) = 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 + \frac{35}{243}x^4$$

$$f_5(x) = 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 + \frac{35}{243}x^4 - \frac{91}{729}x^5$$

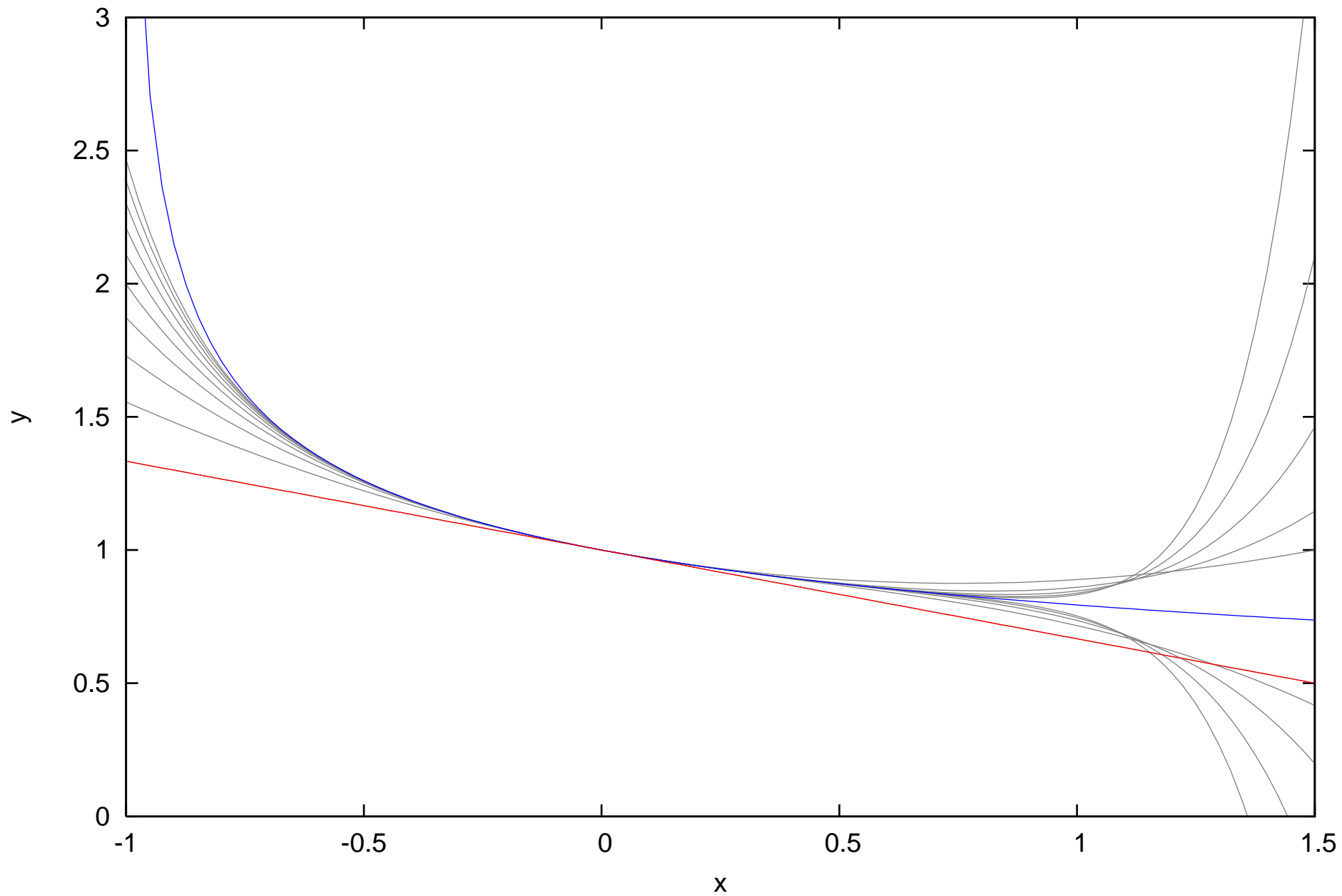
$$f_6(x) = 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 + \frac{35}{243}x^4 - \frac{91}{729}x^5 + \frac{728}{6561}x^6$$

$$f_7(x) = 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 + \frac{35}{243}x^4 - \frac{91}{729}x^5 + \frac{728}{6561}x^6 - \frac{1976}{19683}x^7$$

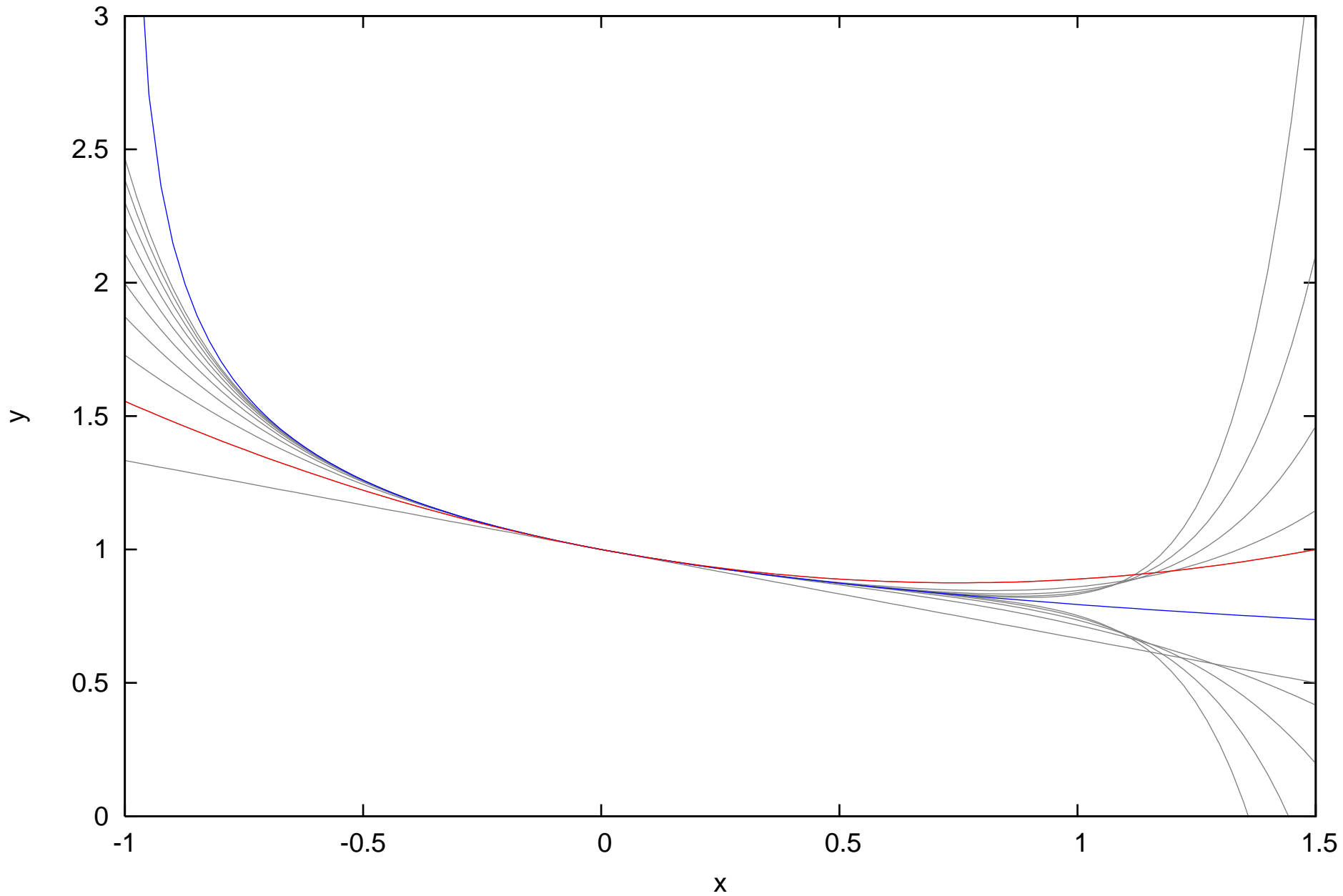
...

【注】  $(1 + x)^{-1/3}$  のテーラー展開の収束半径は 1 である。即ち、 $|x| < 1$  で  $f_n(x) \rightarrow f(x)$  ( $n \rightarrow \infty$ ) となるが、 $|x| > 1$  では  $f_n(x)$  は  $n \rightarrow \infty$  で発散する。

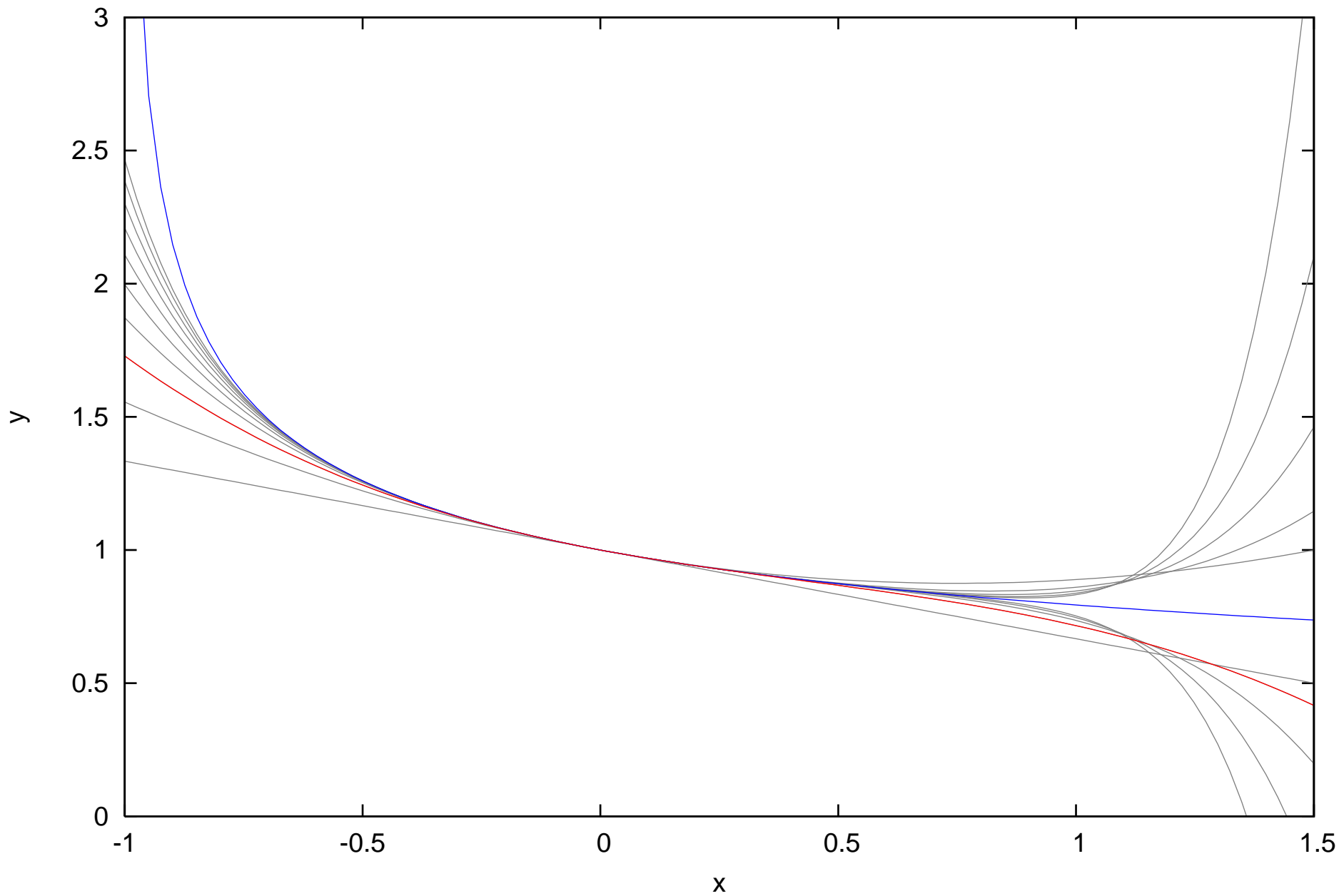
blue curve: $f(x)=(1+x)^{-1/3}$ , red curve: $f_1(x)=1-x/3$ , gray curves: $f_2(x),f_3(x),\dots,f_{10}(x)$



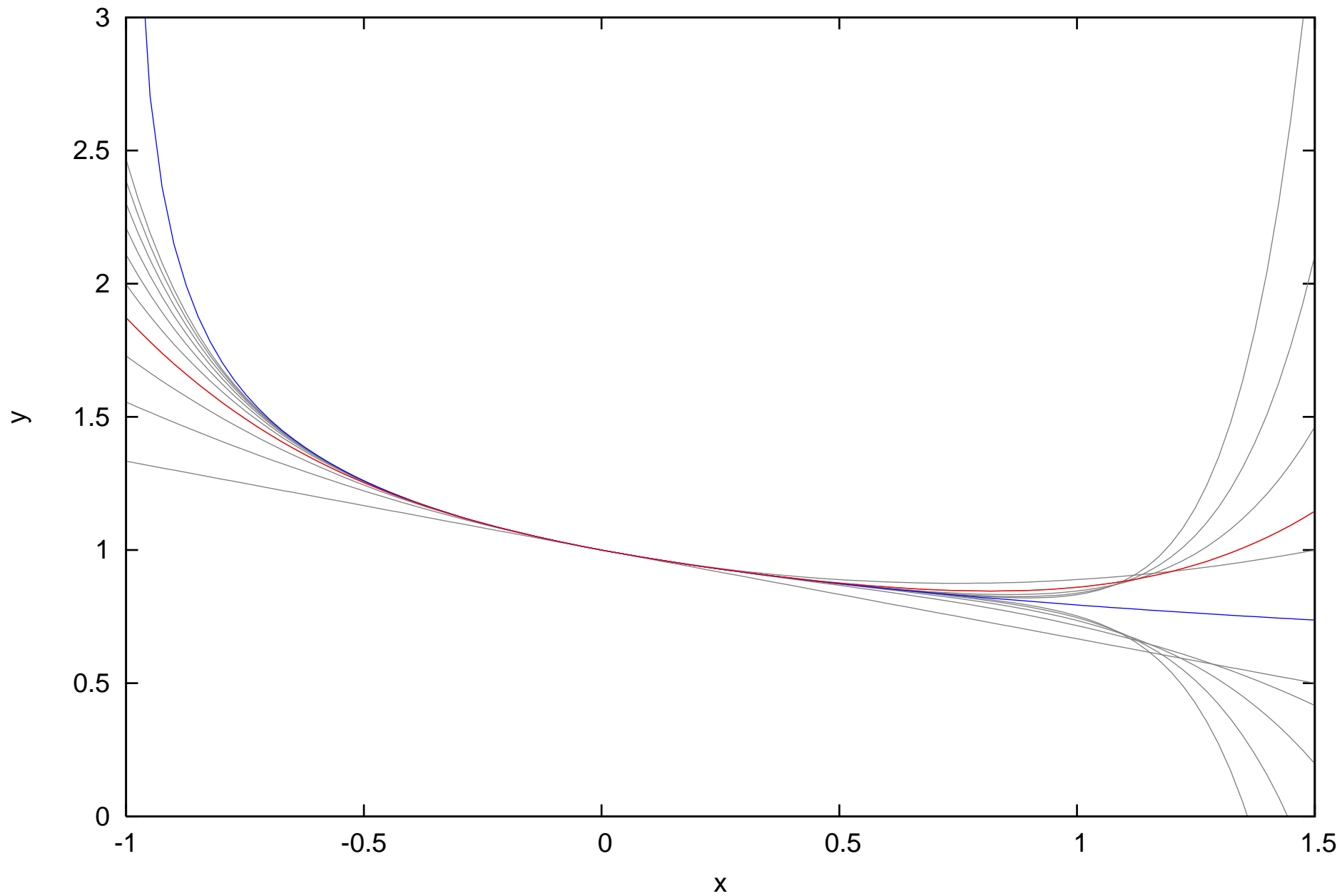
blue curve:  $f(x)=(1+x)^{-1/3}$ , red curve:  $f_2(x)=1-x/3+x^2*2/9$



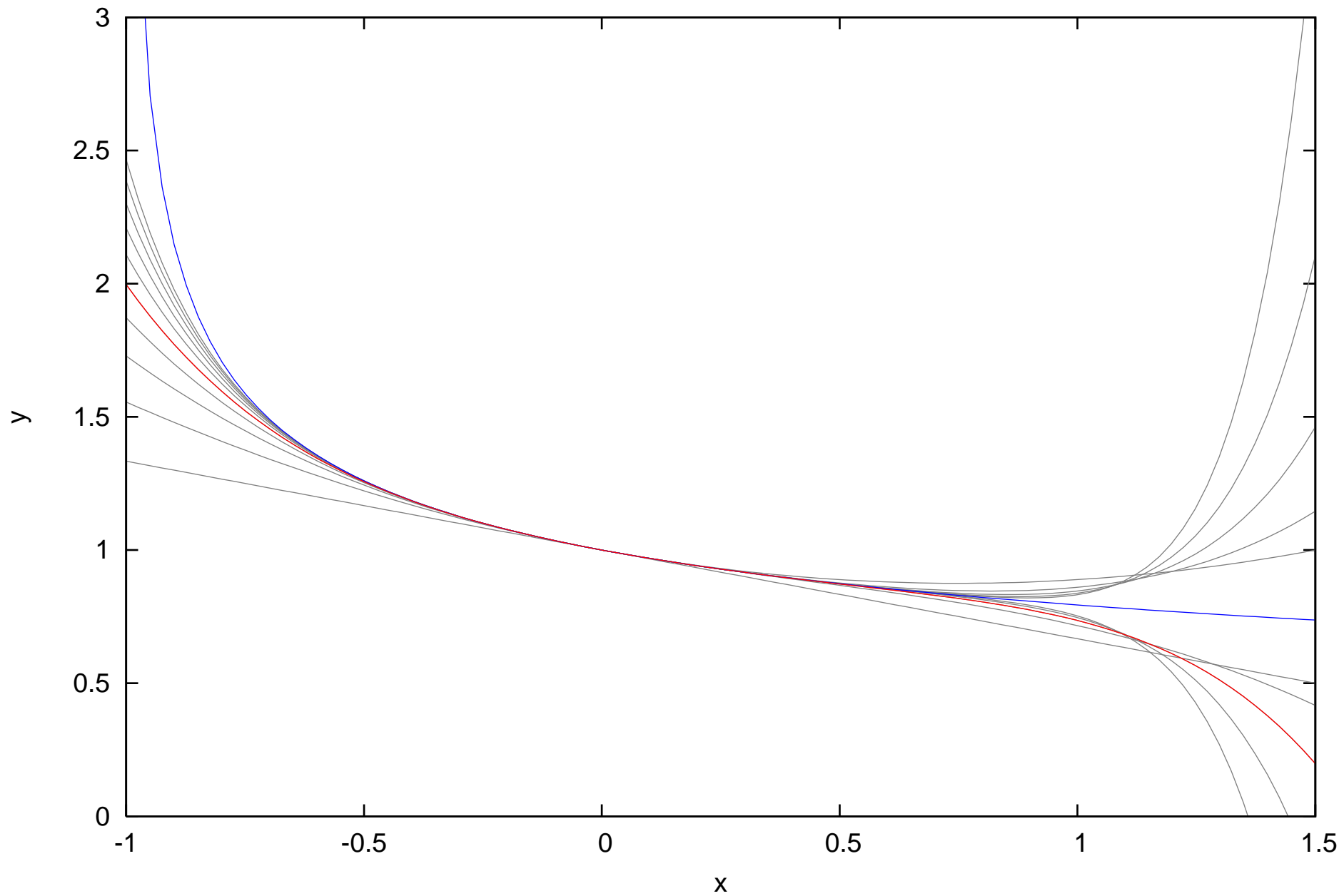
blue curve:  $f(x)=(1+x)^{-1/3}$ , red curve:  $f_3(x)=1-x/3+x^2/8-x^3/81$



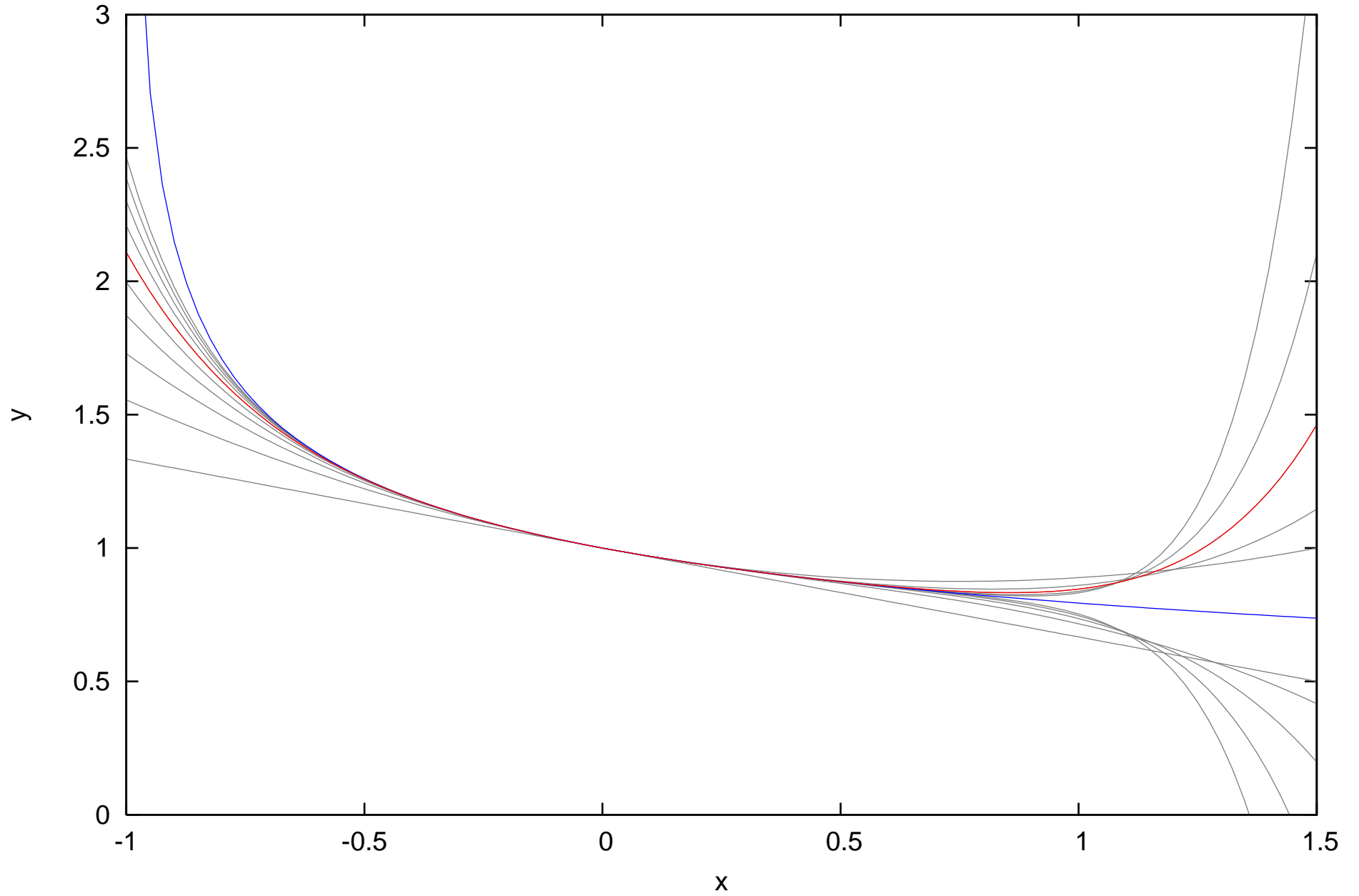
blue curve: $f(x)=(1+x)^{-1/3}$ , red curve: $f_4(x)$



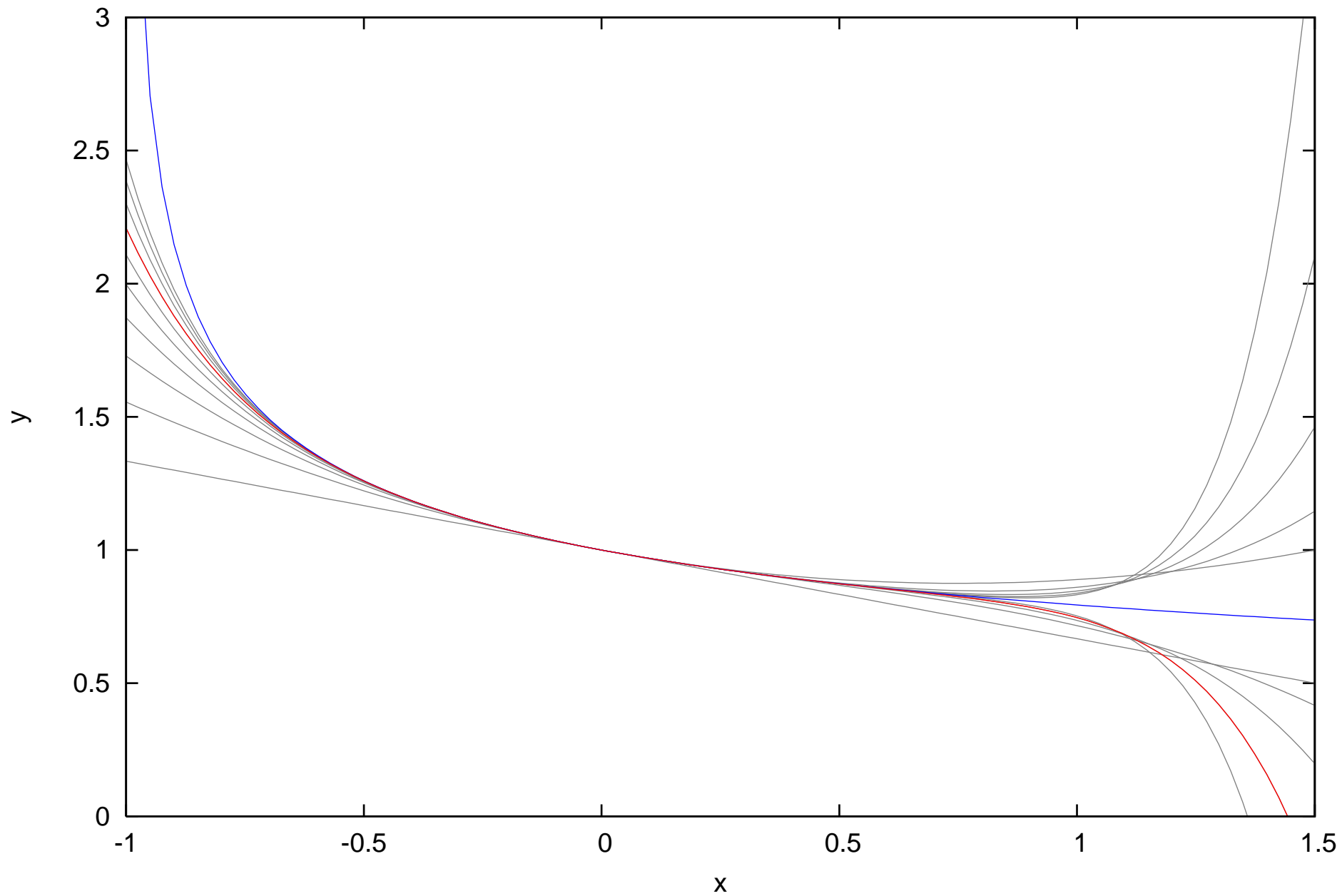
blue curve: $f(x)=(1+x)^{-1/3}$ , red curve: $f_5(x)$



blue curve: $f(x)=(1+x)^{-1/3}$ , red curve: $f_6(x)$

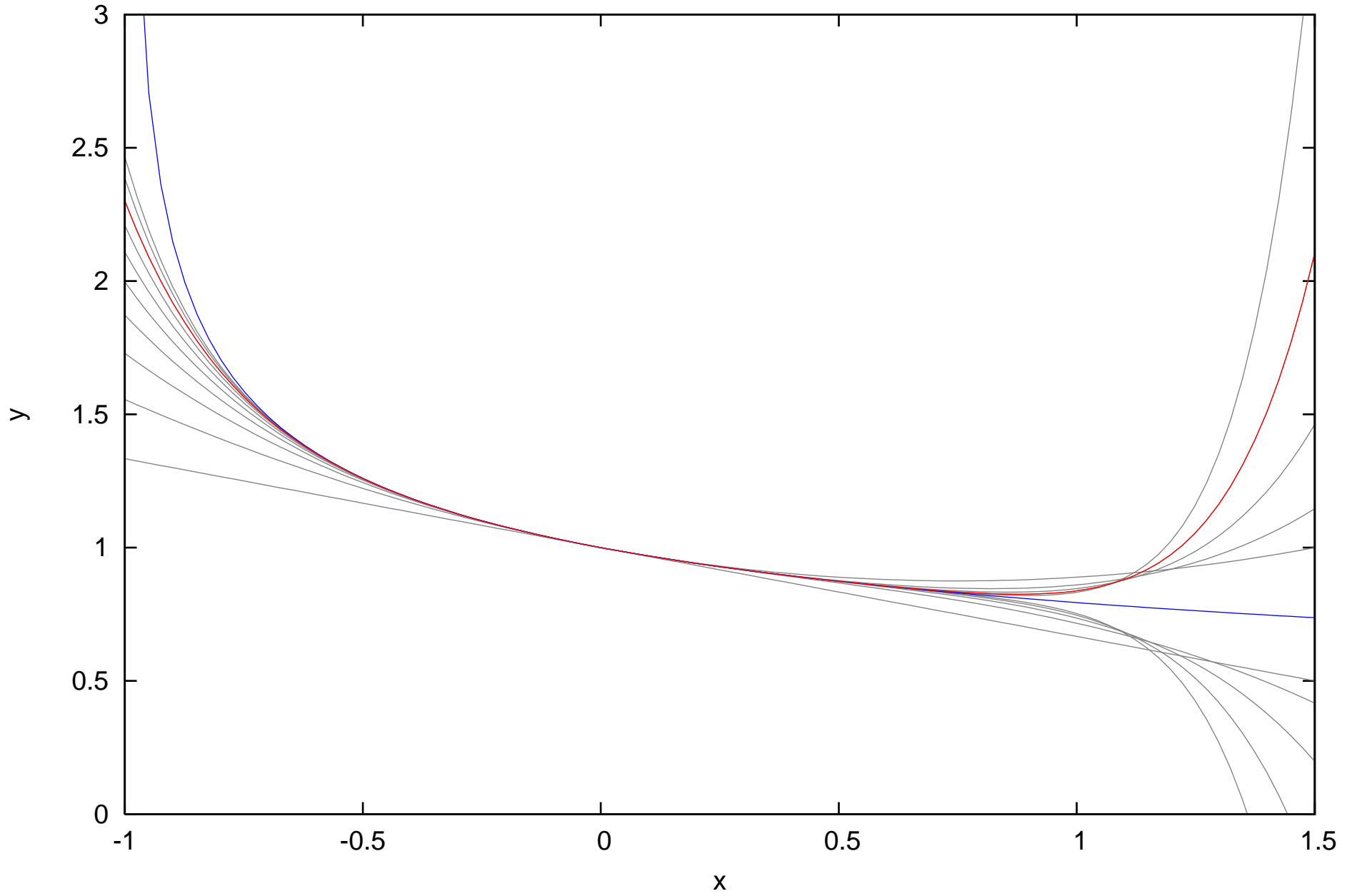


blue curve: $f(x)=(1+x)^{-1/3}$ , red curve: $f_7(x)$

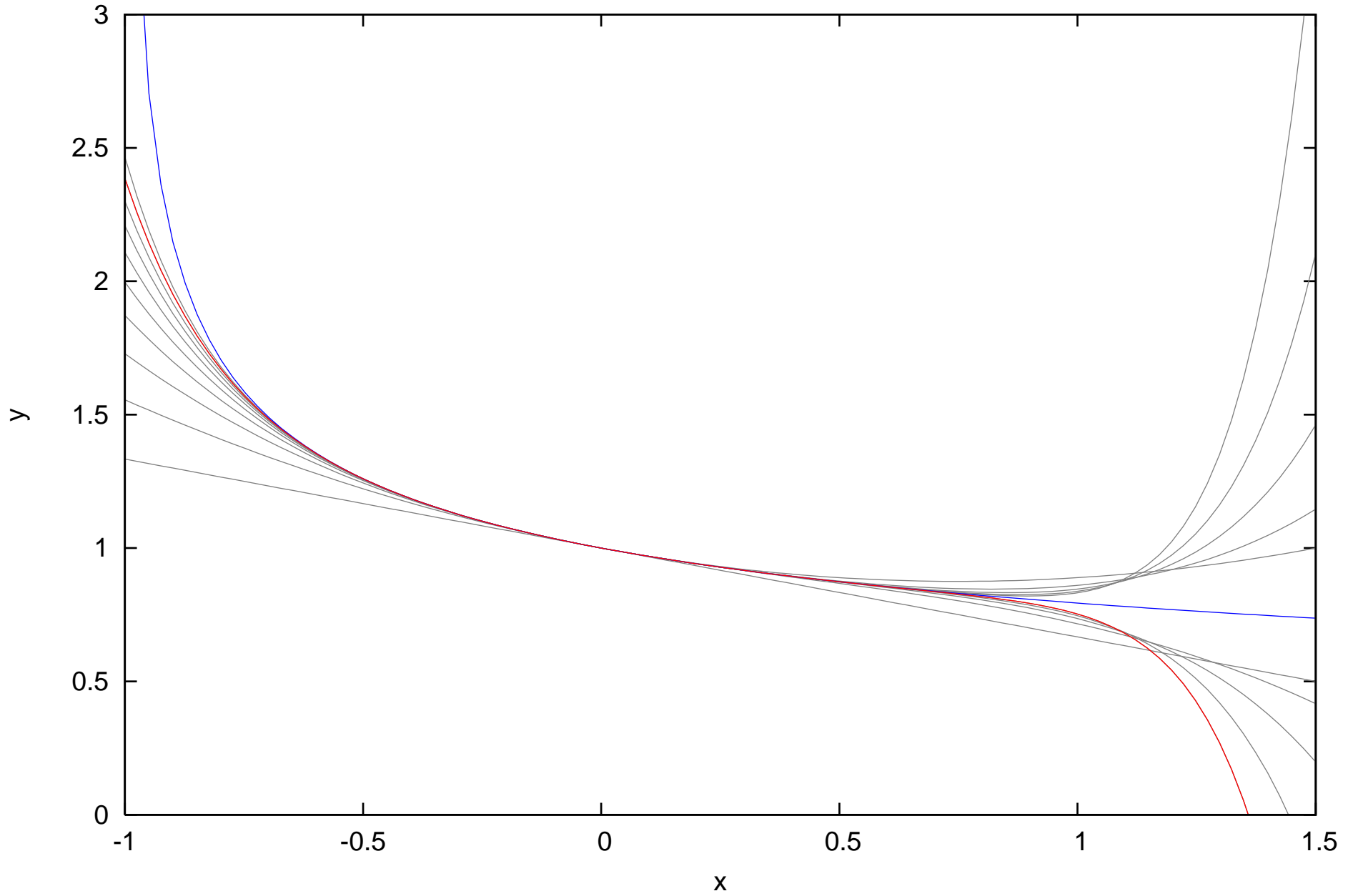




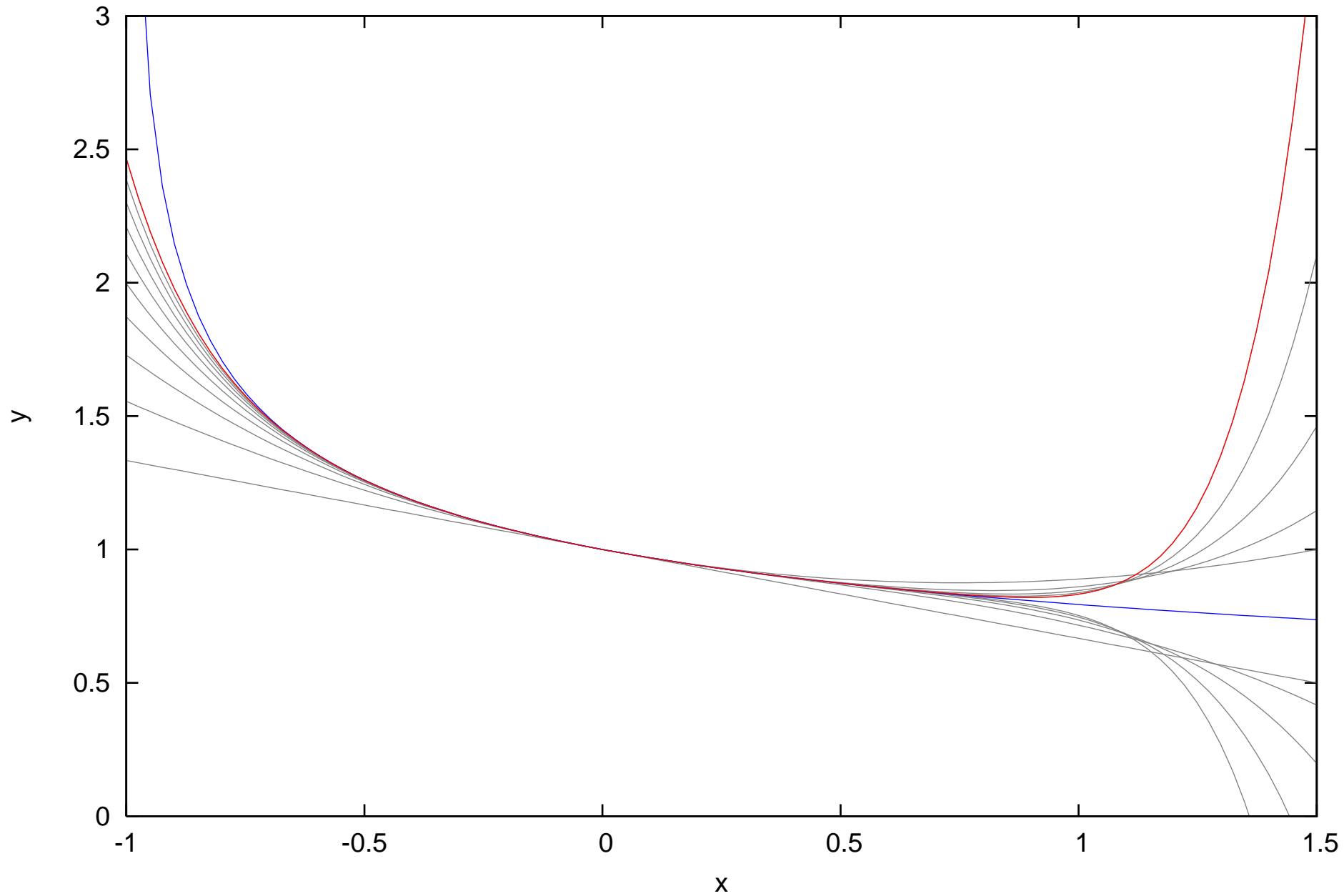
blue curve: $f(x)=(1+x)^{-1/3}$ , red curve: $f_8(x)$



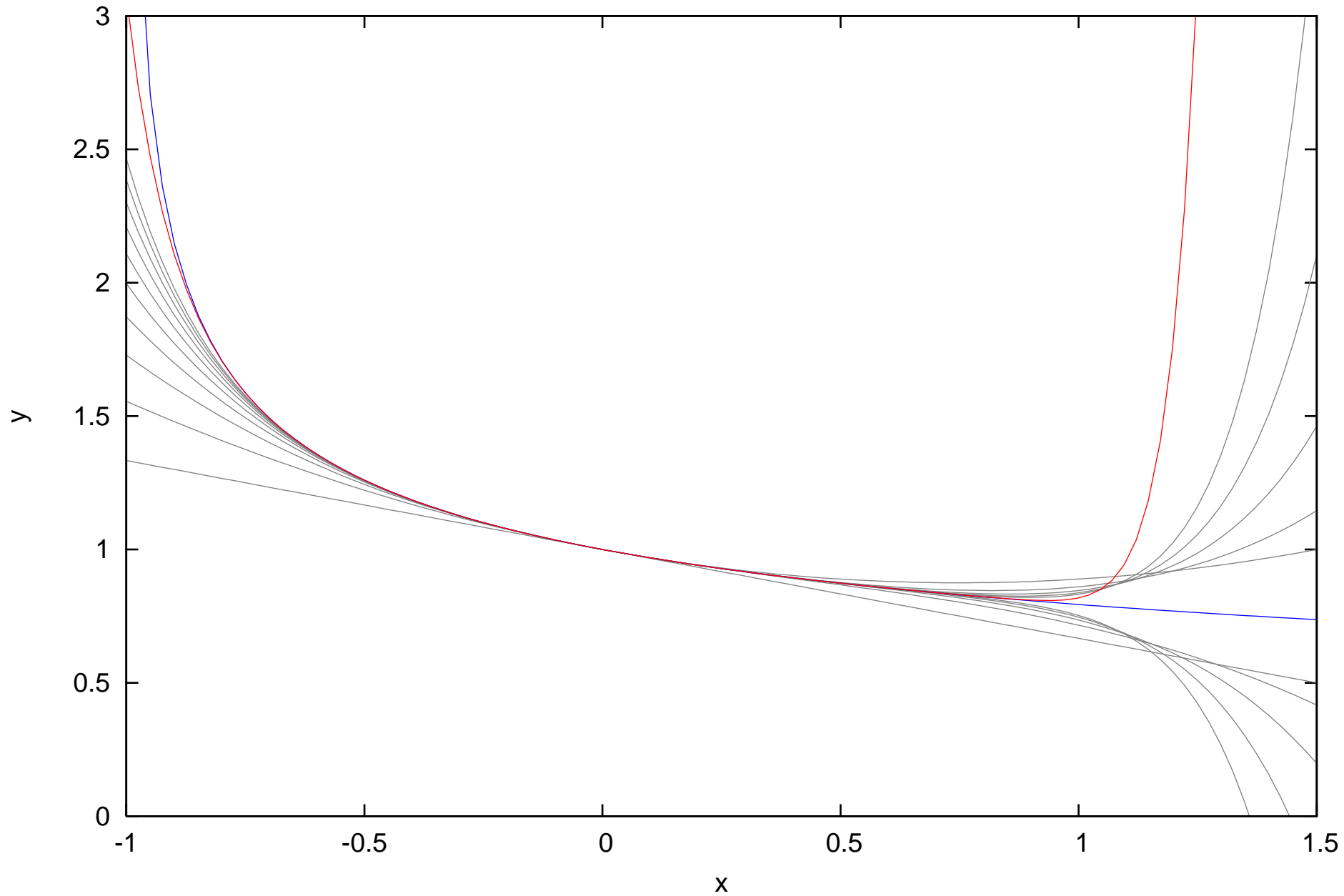
blue curve: $f(x)=(1+x)^{-1/3}$ , red curve: $f_9(x)$



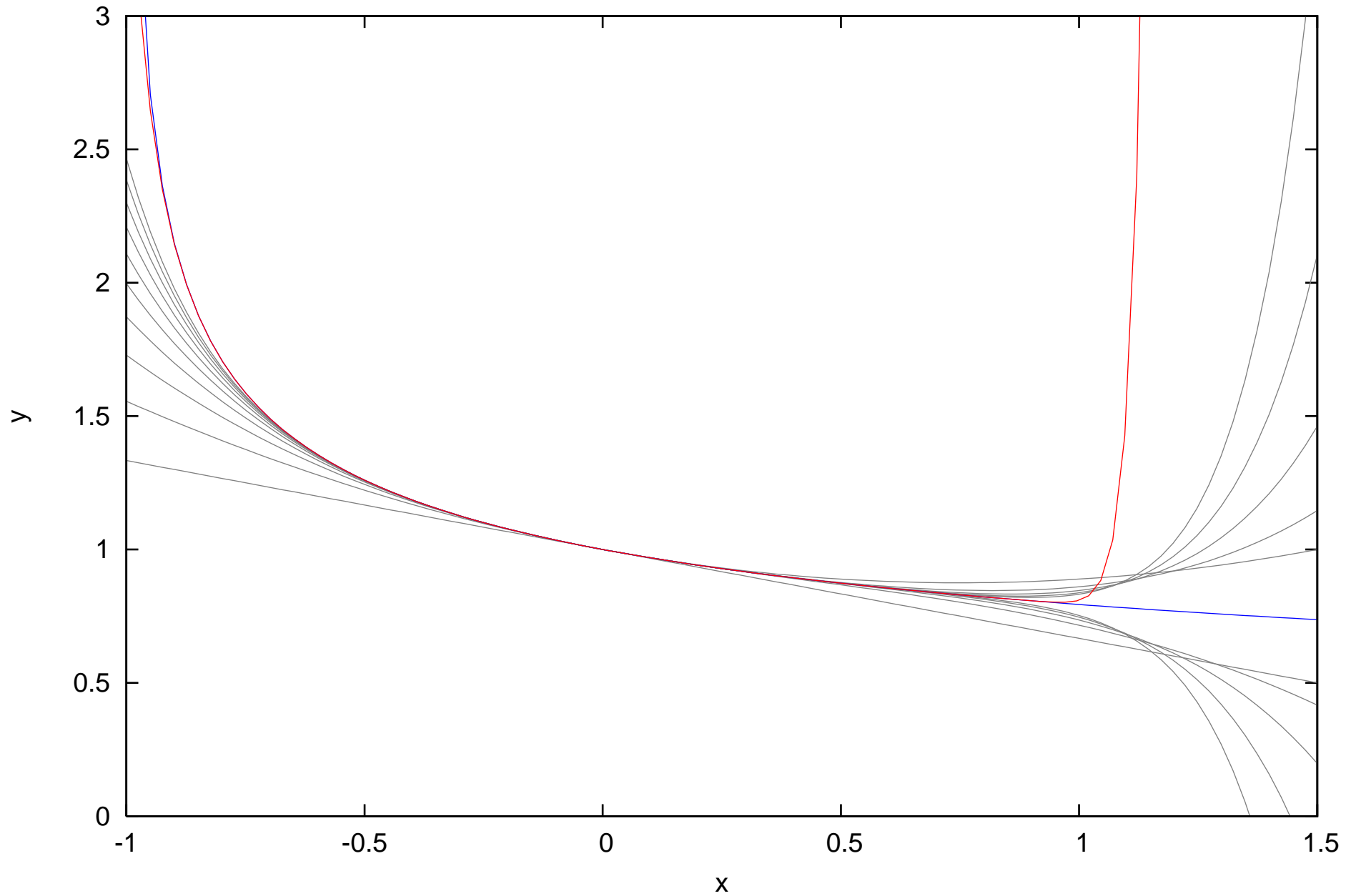
blue curve: $f(x)=(1+x)^{-1/3}$ , red curve: $f_{10}(x)$



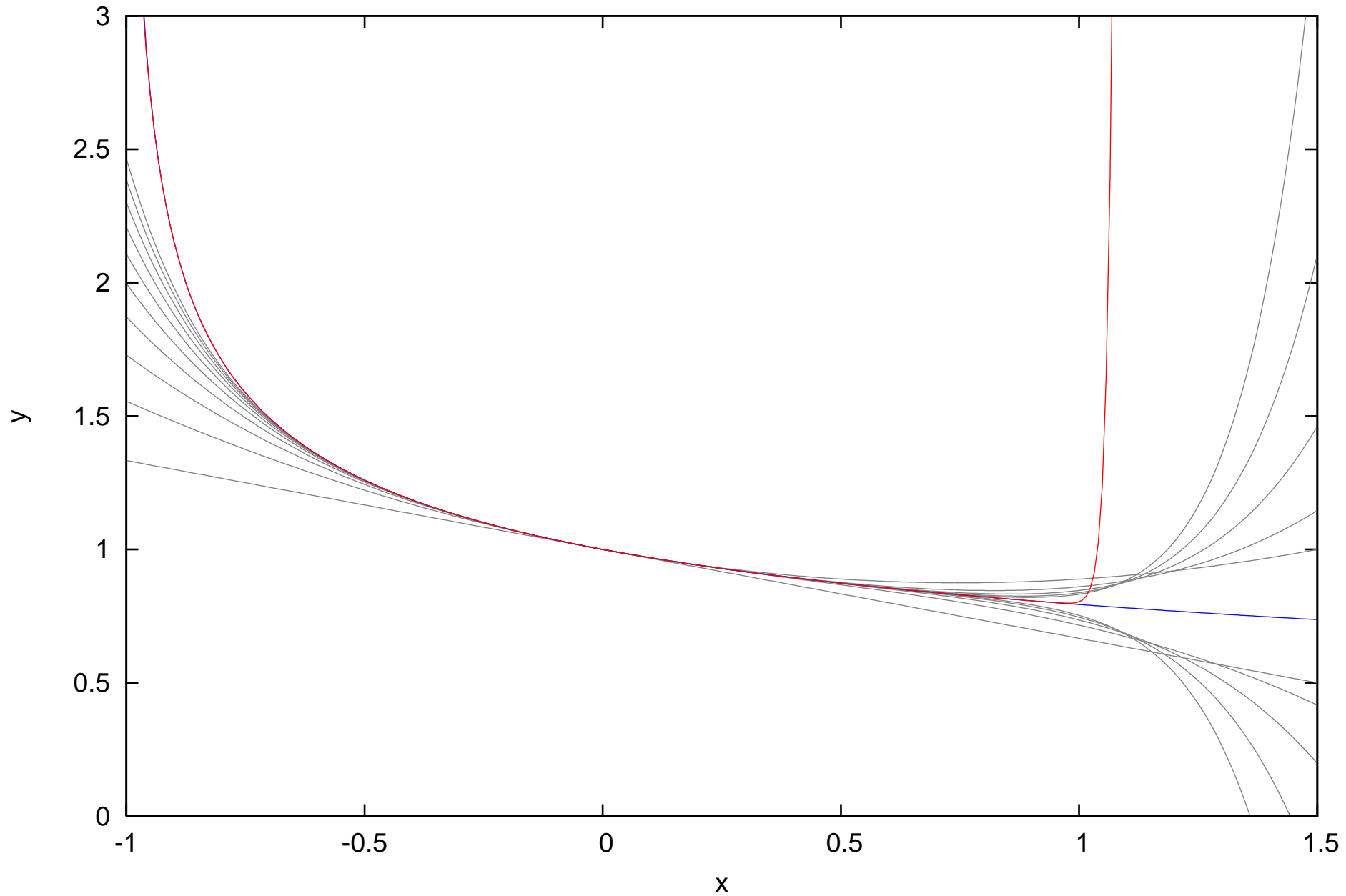
blue curve: $f(x)=(1+x)^{-1/3}$ , red curve: $f_{20}(x)$



blue curve: $f(x)=(1+x)^{-1/3}$ , red curve: $f_{40}(x)$



blue curve: $f(x)=(1+x)^{-1/3}$ , red curve: $f_{80}(x)$



blue curve: $f(x)=(1+x)^{-1/3}$ , red curve: $f_{81}(x)$

