2005. 9. 20, Varna

Sum Rules in Relativistic Nuclear Models
-- Roles of Anti-nucleon Degrees of Freedom --

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Introduction

Main Difference Between Rel. Models and Non-Rel. Models

Anti-nucleons $\bar{N}$

Model-independent Sum Rules
are useful for the study of their Roles.

Naïve Relativistic Correction $v_F^2 \sim 10\ \%$

Correction to Sum Rules comes from the fact that

the Complete Set is composed of

Nucleon and Anti-nucleon Sectors.

If there is correction, we have to study

the coupling of p-h states with N-$\bar{N}$ states.

Vacuum polarization

If there is Divergence , we have to study

Renormalization.

1. Gamow-Teller Sum Rule **Model-independent**

1-1 Non-Relativistic Models

1-2 Relativistic Models

1) Mean Field Approximation

Naïve Relativistic Correction

2) RPA

Coupling with $N\bar{N}$ states

Renormalization of the Divergence

2. Dirac Sea Effects on Other Quantities

3. Conclusions

Challenge in the Future

1 Gamow-Teller Sum Rule

1-1 Non-relativistic models

$$Q_{\pm} = \sum_i^A \tau_{\pm i}\sigma_{yi} \qquad \tau_{\pm} = (\tau_x \pm i\tau_y)/\sqrt{2} \quad , \qquad \tau_0 = \tau_z .$$

Sum Rule for β_- and β_+ transitions

$$\sum_n \{\langle\ | Q_+ | n\rangle\langle n | Q_- |\ \rangle - \langle\ | Q_- | n\rangle\langle n | Q_+ |\ \rangle\}$$

$$= \langle\ | [Q_+, Q_-] |\ \rangle = 2(N - Z),$$

3

because

$$[\tau_+\sigma_y\,,\ \tau_-\sigma_y] = 2\tau_z .$$

(Ikeda-Fujii-Fujita Sum Rule).

If we assume

$$Q_+|\ \rangle = 0,$$

we have

$$\langle\ | Q_+ Q_- |\ \rangle = 2(N - Z).$$

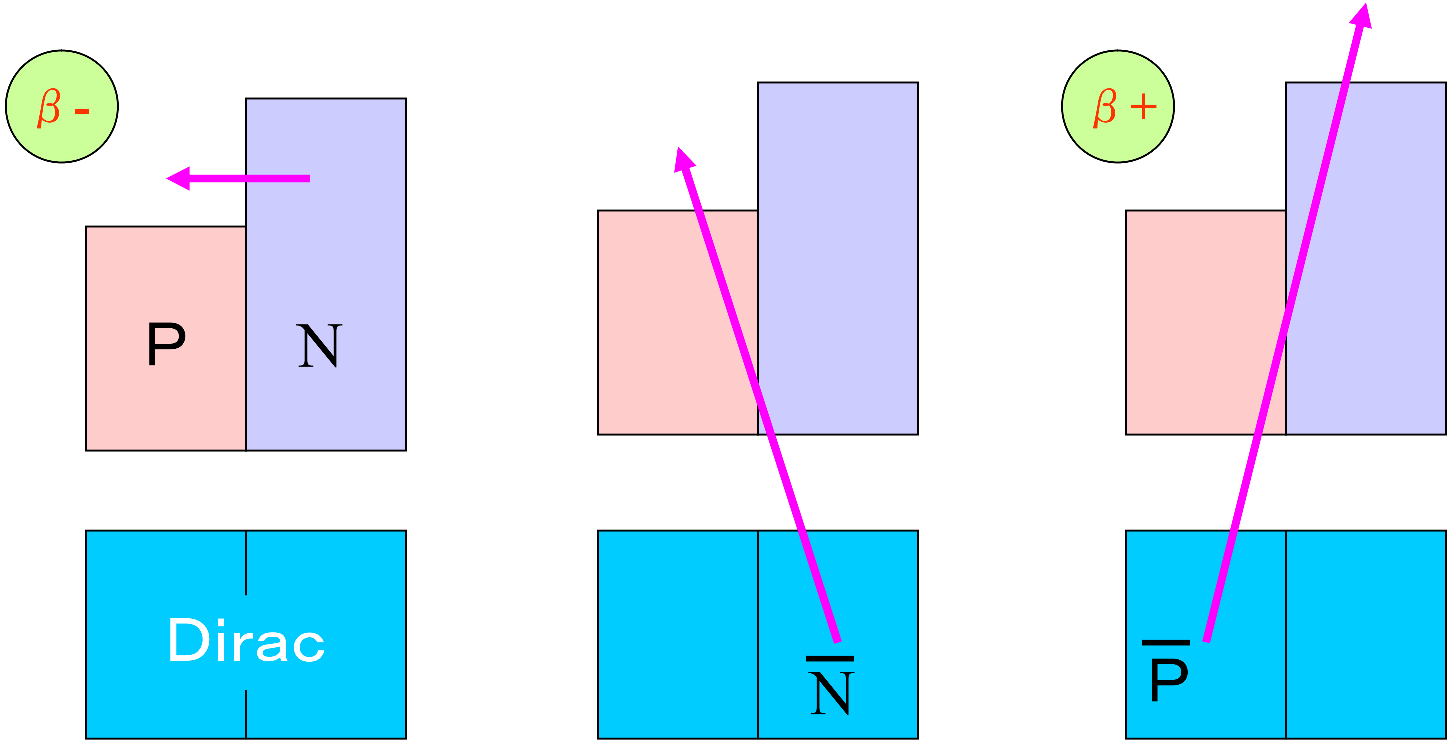
1-2 Relativistic Models

$N > Z$ $\Gamma = \gamma_5 \gamma_y \tau_\pm$

1) Mean Field Approximation

Naïve Relativistic Correction

$$A \quad + \quad (\infty - B) \quad - \quad (\infty - C)$$



Naïve Correction

Relativistic Models for Nuclear Matter

The field:

$$\psi(\boldsymbol{x}) = \int \frac{d^3p}{(2\pi)^{3/2}} \sum_{\alpha} (u_\alpha(\boldsymbol{p}) \exp(i\boldsymbol{p}\cdot\boldsymbol{x})\, a_\alpha(\boldsymbol{p}) + v_\alpha(\boldsymbol{p}) \exp(-i\boldsymbol{p}\cdot\boldsymbol{x})\, b_\alpha^\dagger(\boldsymbol{p})),$$

$$u_\alpha(\boldsymbol{p}) = u_\sigma(\boldsymbol{p})\,|\tau\rangle \quad (\ \alpha = \sigma, \tau\ ), \quad \textbf{etc.}$$

Positive and Negative spinor:

$$u_\sigma(\boldsymbol{p}) = \left[\frac{E_p + M^*}{2E_p}\right]^{1/2} \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E_p+M^*} \end{pmatrix} \xi\,, \qquad v_\sigma(\boldsymbol{p}) = \left[\frac{E_p + M^*}{2E_p}\right]^{1/2} \begin{pmatrix} \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E_p+M^*} \\ 1 \end{pmatrix} \xi$$

$E_p = \sqrt{M^{*2} + \boldsymbol{p}^2}$, ξ **: Pauli spinor** M^* **: the nucleon effective mass**

M* = M – Us (Lorentz scalar Potential)

The matrix elements for the β_- and β_+ transitions:

β -

$$\langle \,|\, F_+ F_- \,|\, \rangle = 4 \frac{V}{(2\pi)^3} \int \frac{d^3p}{E_p^2} \left[\theta_{\boldsymbol{p}}^{(\mathrm{n})} \left(1 - \theta_{\boldsymbol{p}}^{(\mathrm{p})}\right) \left(M^{*2} + p_y^2\right) + \left(1 - \theta_{\boldsymbol{p}}^{(\mathrm{p})}\right) \left(\boldsymbol{p}^2 - p_y^2\right) \right],$$

β +

$$\langle \,|\, F_- F_+ \,|\, \rangle = 4 \frac{V}{(2\pi)^3} \int \frac{d^3p}{E_p^2} \left(1 - \theta_{\boldsymbol{p}}^{(\mathrm{n})}\right) \left(\boldsymbol{p}^2 - p_y^2\right).$$

Infinite

$\theta_{\boldsymbol{p}}^{(i)} = \theta(k_i - |\boldsymbol{p}|)$ for i =p and n, $\quad V = A\left(3\pi^2/2k_{\mathrm{F}}^3\right)$,
k_{n} and k_{p} : Fermi momentum of neutrons and protons

Each of them is ~~divergent.~~ **infinite.**
However, the difference gives the sum rule value;

$$\langle \,|\, F_+ F_- \,|\, \rangle - \langle \,|\, F_- F_+ \,|\, \rangle = 2(N - Z).$$

The anti-nucleon space is necessary.

Pauli blocking terms give Rel. correction.

For the nucleon degrees of freedom only,

$$\langle \, | F_+ F_- | \, \rangle \approx 2 \left(1 - \frac{2}{3} v_F^2 \right) (N - Z)$$

Naïve Rel. Correction
(N-Z)-expansion

v_F : Fermi velocity

The classical sum value is quenched by 12%

for $v_F = 0.43 \quad (M^* = 0.6M, \quad k_F = 1.36 \mathrm{fm}^{-1})$.

The GT strength taken by N$\bar{\mathrm{N}}$ excitations in the time-like rgion:

$$[\langle \, | F_+ F_- | \, \rangle - \langle \, | F_- F_+ | \, \rangle]_{\mathrm{N}\bar{\mathrm{N}}} \approx 2 \, \frac{2}{3} v_F^2 \, (N - Z)$$

H. Kurasawa, T. Suzuki and N. V. Giai, Phys. Rev. Lett. 91, 062501 (2003),
H. Kurasawa, T. Suzuki and N. V. Giai, Phys. Rev. C68, 064311 (2003)
H. Kurasawa and T.Suzuki, Phys. Rev. C69, 014306 (2004)

GT strengths in ^{48}Ca

14.997 / 16 = 0.937

6.3%

$n \to p$

$n\ell j$	$n'\ell'j'$	$T_{aa'}$	$g(a:a')$	$f(a:a')$
$1p_{3/2}(-39.41)$	$2p_{3/2}(-1.09)$	0.000	−0.0113	−0.0116
$1p_{3/2}(-39.41)$	$1f_{5/2}(-1.16)$	0.002	0.8084	−0.0180
$1p_{1/2}(-36.23)$	$2p_{3/2}(-1.09)$	0.005	−0.0360	0.0117
$1f_{7/2}(-10.00)$	$1f_{5/2}(-1.16)$	8.629	0.9715	−0.0148
$1f_{7/2}(-10.00)$	$1f_{7/2}(-9.59)$	6.361	0.9787	0.0200
Total		14.997		

H.Kurasawa, T. Suzuki and N.V. Giai, Phys. Rev. C68, 064311(2003) , NL-SH

^{90}Zr 7.7%

^{208}Pb 8.4%

Z.Y. Ma, B.Q. Chen N.V. Giai and T. Suzuki,
Eur.Phys. J. A20, 429 (2004) , NL3

M* in finite nuclei is larger than in nuclear matter.

Relativistic correction is not negligible.
$6 \sim 8$ % quenching

This means that

the coupling of the p-h states and $N\bar{N}$ states should be examined.

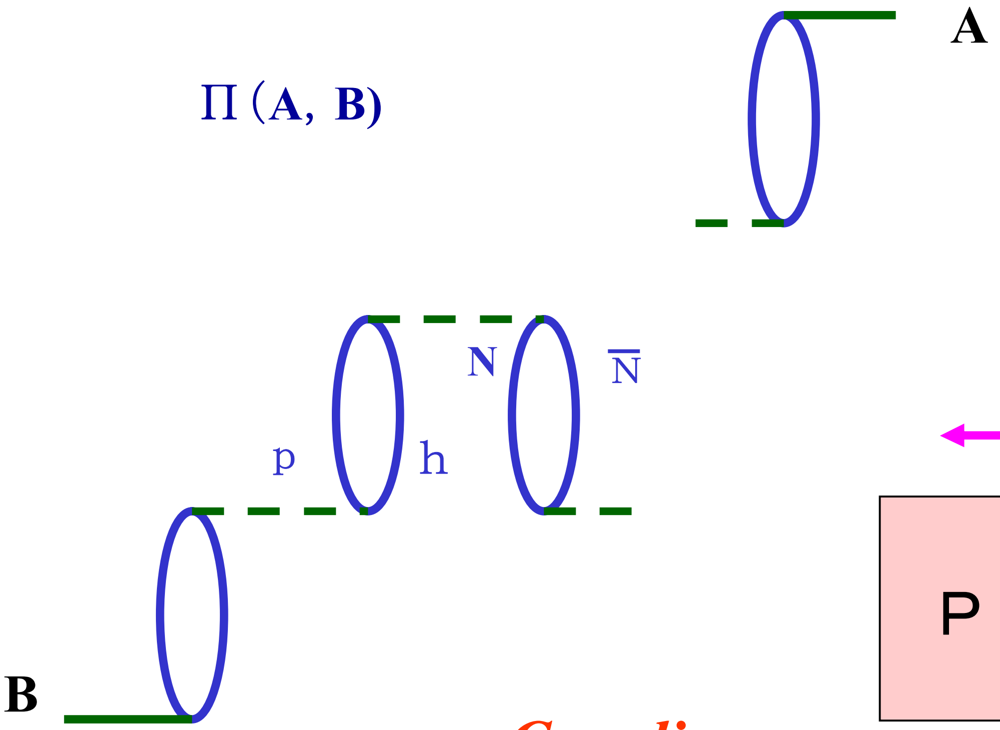
Because

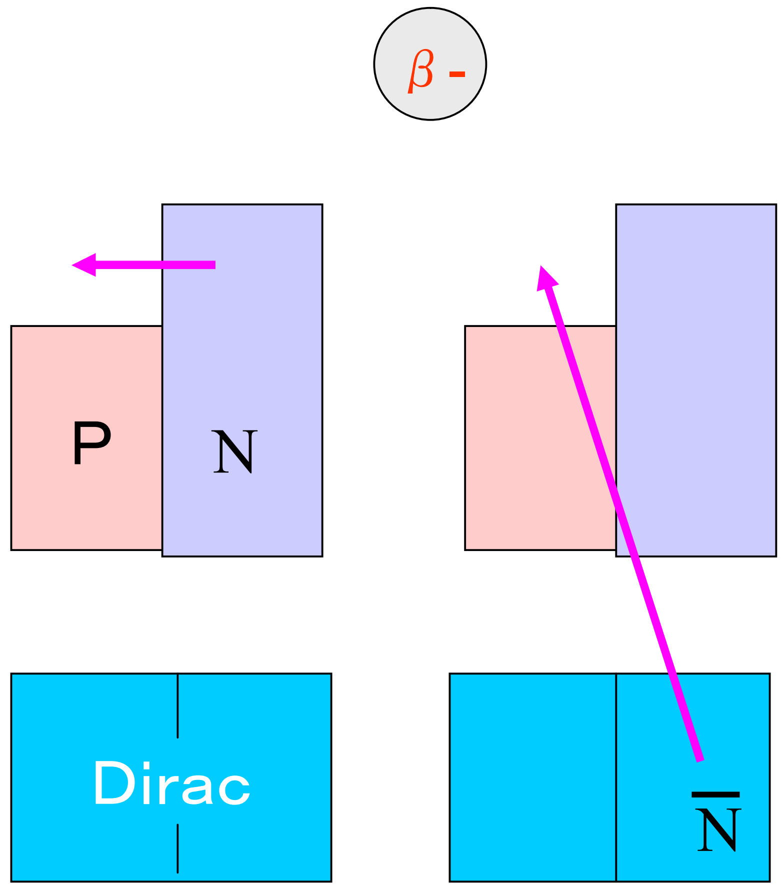
1) The total strength of $N\bar{N}$ is infinite.

2) Recent Exp. has observed $12 \pm 6 \pm 5$ % quenching.

K. Yako et al., Phys. Lett. B615 (2005) 193

2) RPA with Renormalization in Nuclear Matter





Coupling

Model :

① **Interaction**

② **Renormalization**

① Interaction

Non-Rel.

$$V = \left[\left(\frac{f_\pi}{m_\pi}\right)^2 g' \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \left(\frac{f_\pi}{m_\pi}\right)^2 \frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q})}{\boldsymbol{q}^2 + m_\pi^2} - \left(\frac{f_\rho}{m_\rho}\right)^2 \frac{(\boldsymbol{\sigma}_1 \times \boldsymbol{q})(\boldsymbol{\sigma}_2 \times \boldsymbol{q})}{\boldsymbol{q}^2 + m_\rho^2}\right] \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

Rel.

$$\mathcal{L} = \frac{g_a}{2} \overline{\psi} \Gamma_i^\mu \psi \, \overline{\psi} \Gamma_{\mu i} \psi + \frac{g_t}{4} \overline{\psi} T_i^{\mu\nu} \psi \, \overline{\psi} T_{\mu\nu\, i} \, \psi,$$

$$\Gamma_i^\mu = \gamma_5 \gamma^\mu \tau_i \,, \quad T_i^{\mu\nu} = \sigma^{\mu\nu} \tau_i, \qquad g_a, \quad -g_t \to \left(\frac{f_\pi}{m_\pi}\right)^2 g'$$

T. Maruyama, H. Kurasawa, and T. Suzuki, nucl-th/0404074l ; Prog. Theor. Phys. 113, 355 (2005)

Excitation Energy and Strength of the GT state

$$\omega_{\rm GT} \approx \frac{4k_{\rm F}^3}{3\pi^2}\frac{g_a}{1+2\kappa'(\bar{N})g_a/(2\pi)^3}\left(1-\frac{2}{3}v_{\rm F}^2\right)\frac{N-Z}{A}$$

$\overline{\rm N}$N Effect

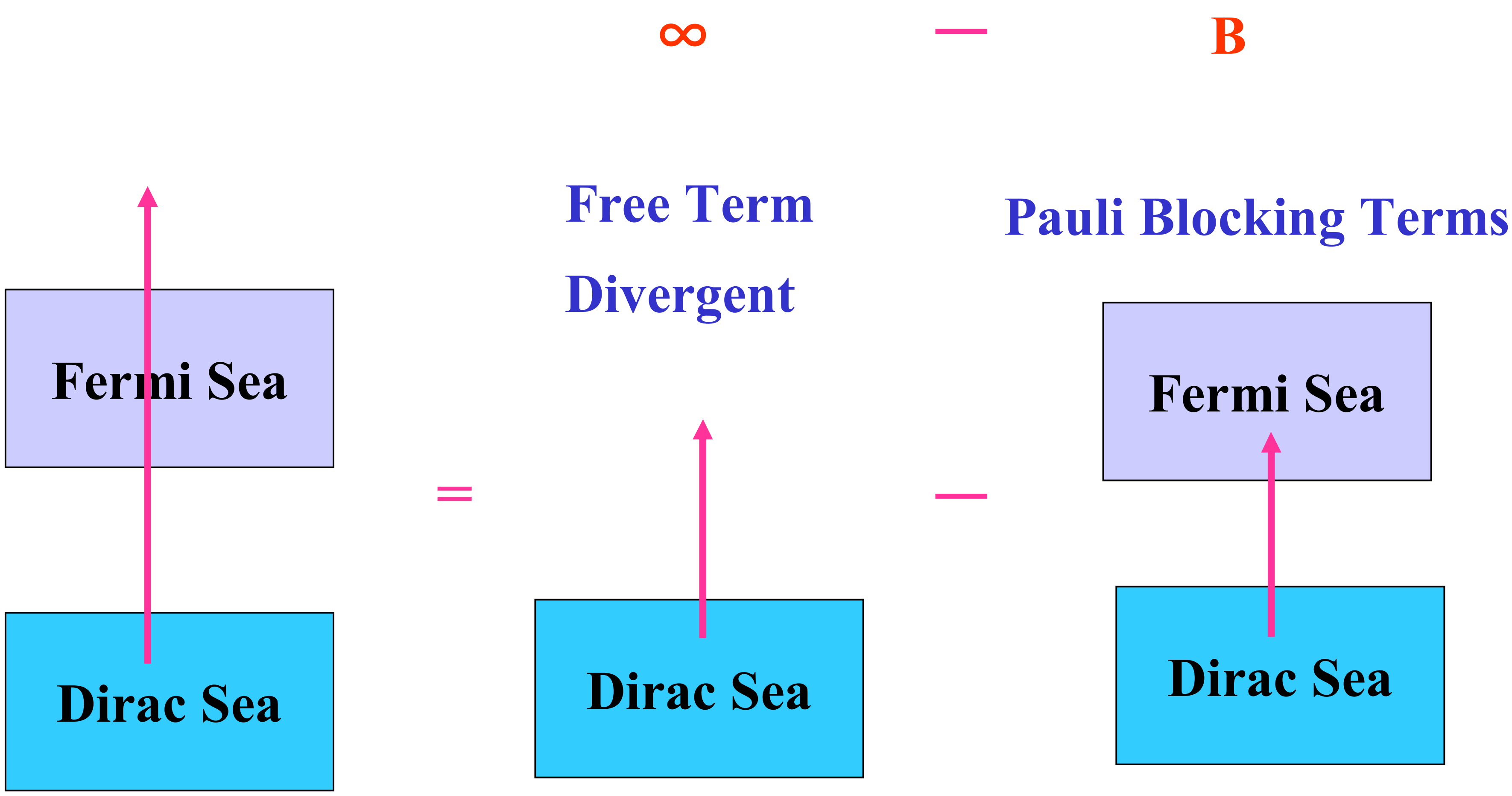
Naïve Rel. Correction

$$S_- \approx \frac{1}{\left(1+2\kappa'(\bar{N})g_a/(2\pi)^3\right)^2}\left(1-\frac{2}{3}v_{\rm F}^2\right)2\,(N-Z)$$

$$\kappa'(\bar{N}) = \frac{2}{3}\int d^3p\,\frac{\boldsymbol{p}^2}{E_{\boldsymbol{p}}^3}\left(2-\theta_{\boldsymbol{p}}^{(\rm n)}-\theta_{\boldsymbol{p}}^{(\rm p)}\right) > 0$$

Divergence

Comment on No-Sea Approximation



Z. Ma, B. Chen, N.V. Giai and T. Suzuki, Eur. Phys. J. A 20, 429(2004)

No-Sea RPA

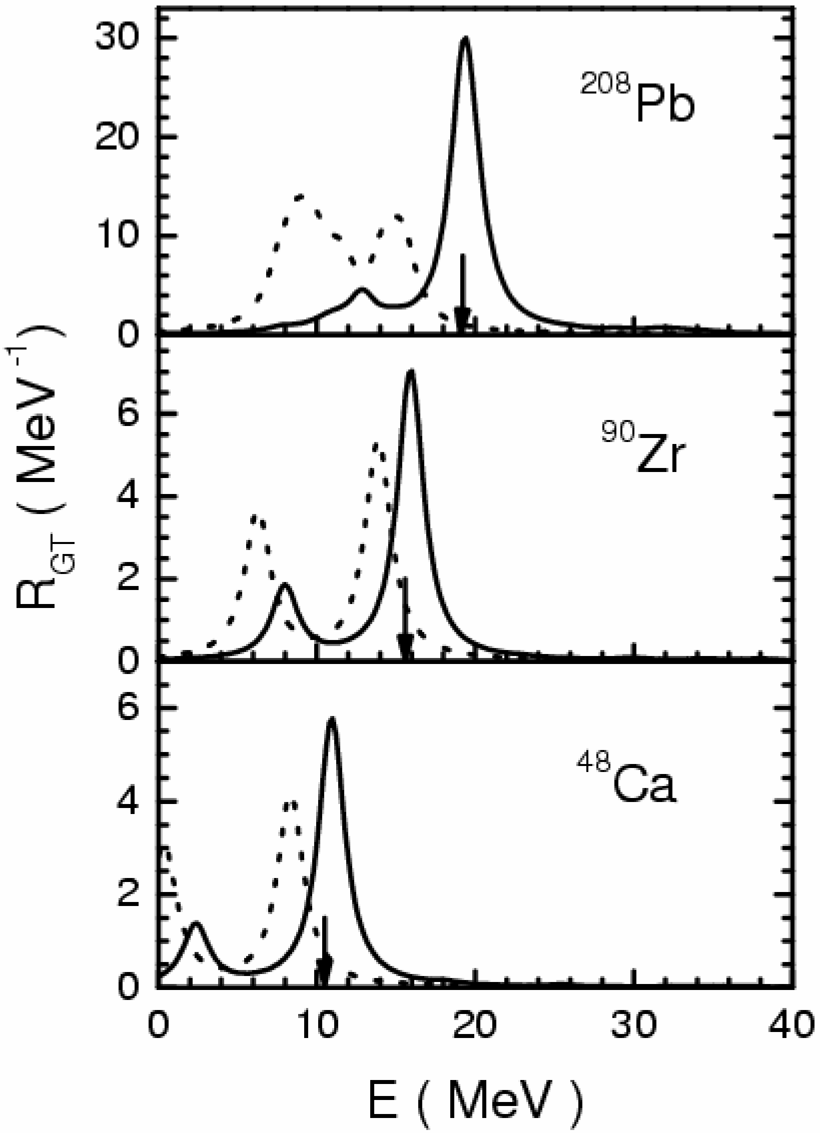
Divergence is neglected.

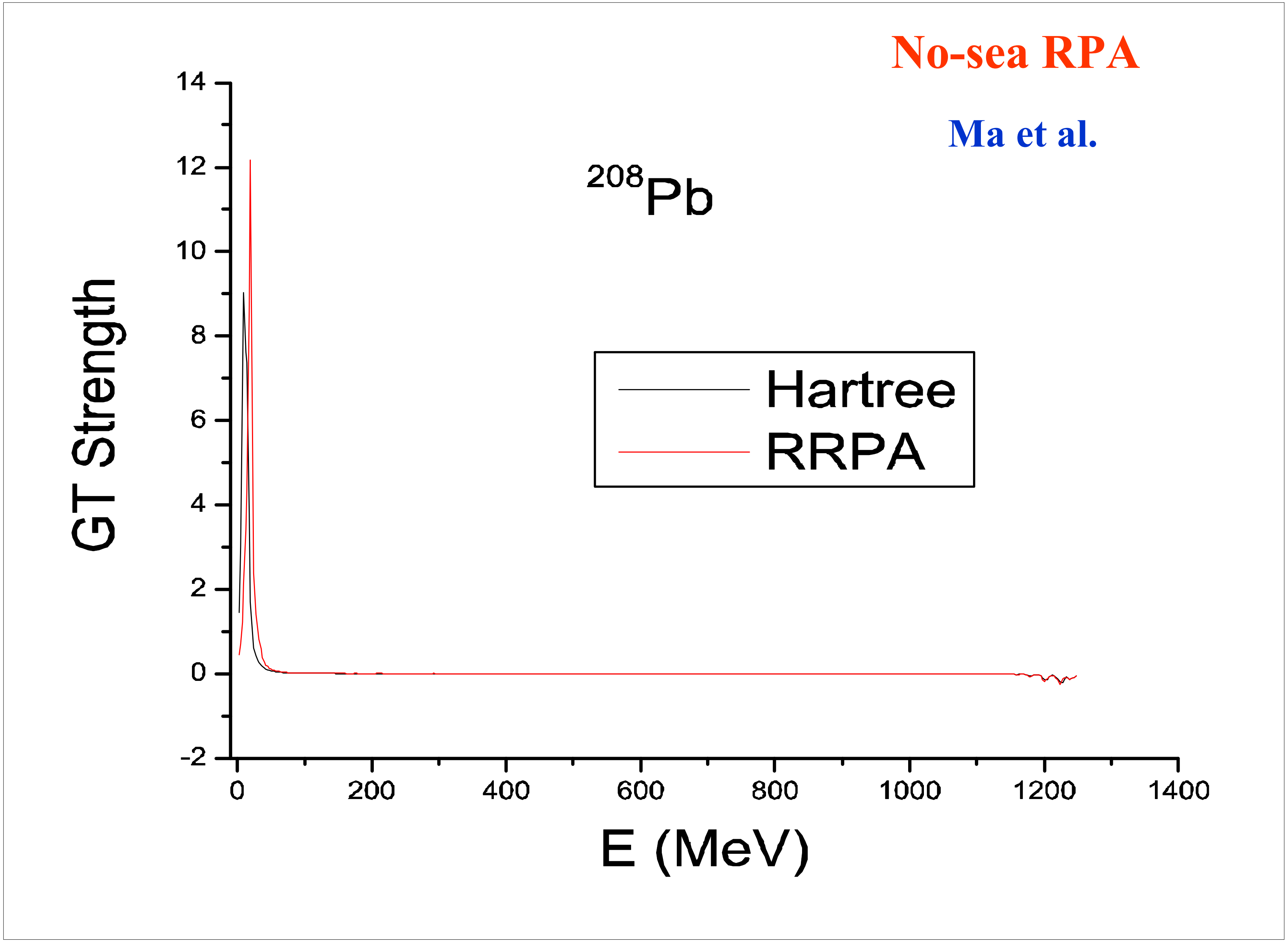
NL3

+ g'+V_π+V_ρ

(g' = 0.6)

94 % Strength

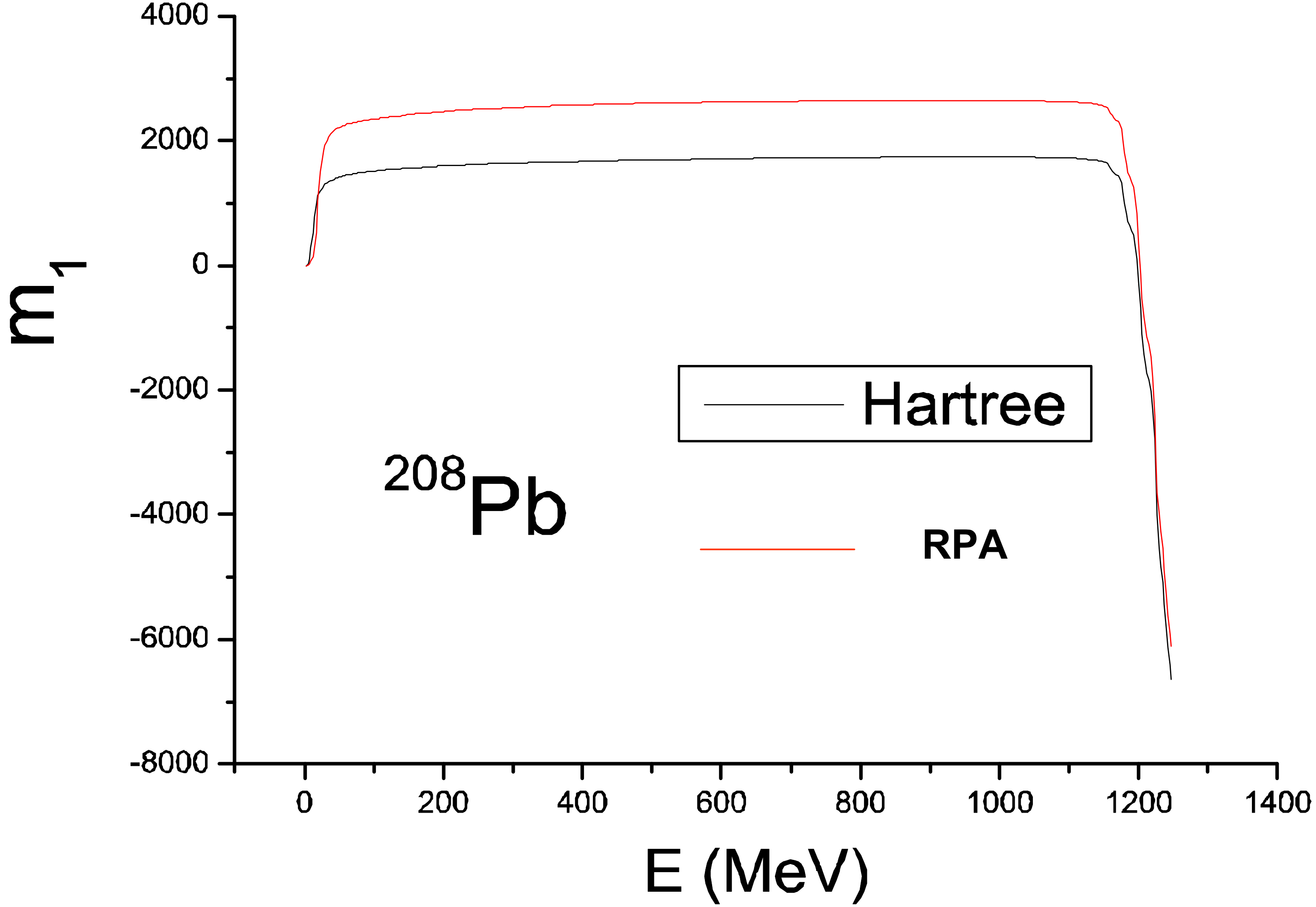



No-sea RPA
Ma et al.
208Pb
Hartree
RRPA
GT Strength
E (MeV)
14
12
10
8
6
4
2
0
-2
0
200
400
600
800
1000
1200
1400

No-sea RPA

Sum of the Energy-weighted Strengths

Ma et al.



Excitation Energy and Strength of the GT state

$$\omega_{\rm GT} \approx \frac{4k_{\rm F}^3}{3\pi^2}\frac{g_a}{1+2\kappa'(\bar{N})g_a/(2\pi)^3}\left(1-\frac{2}{3}v_{\rm F}^2\right)\frac{N-Z}{A}$$

$\overline{\text{N}}$N Effect

Naïve Rel. Correction

$$S_- \approx \frac{1}{\left(1+2\kappa'(\bar{N})g_a/(2\pi)^3\right)^2}\left(1-\frac{2}{3}v_{\rm F}^2\right)2\,(N-Z)$$

$$\kappa'(\bar{N}) = \frac{2}{3}\int d^3p\,\frac{\boldsymbol{p}^2}{E_{\boldsymbol{p}}^3}\left(2-\theta_{\boldsymbol{p}}^{(\rm n)}-\theta_{\boldsymbol{p}}^{(\rm p)}\right) > 0$$

Divergence

② Renormalization method   G. 't Hooft and M. Veltman, N.P.B44 (1972) 189

$$\frac{2}{3}\int d^3p\,\frac{\boldsymbol{p}^2}{E_{\boldsymbol{p}}^3}2 = i\frac{2}{\pi}\int d^4p\frac{M^* + p^2 + 2p_y^2}{(p^2 - M^{*2} + i\epsilon)^2}$$

$$\rightarrow i\frac{2}{\pi}\int d^np\frac{M^* + p^2 + 2p_y^2}{(p^2 - M^{*2} + i\epsilon)^2}$$

$$= -2(2-n)\pi^{n/2-1}M^{*(n-2)}\Gamma(1-n/2)$$

(n ⟶ 4) $$\rightarrow 4\pi\left\{M^{*2}\Gamma(1-n/2) + 2M^{*2}\ln M^*\right\}$$

Medium effect

$$M^{*2} = (M - U_s)^2 = M^2 - 2MU_s + U_s^2$$

$$\Delta\mathcal{L}_c = \frac{1}{2}\left\{a_0 + a_1\sigma + \frac{1}{2}a_2\sigma^2\right\}\boldsymbol{A}_\mu\boldsymbol{A}^\mu$$ Counter Terms

T. Maruyama, H. Kurasawa and T.Suzuki, to be published.
H. Kurasawa and T. Suzuki, Invited Paper , to be published.

$$\kappa'(\bar{N}) = \frac{2}{3}\int d^3p\,\frac{\boldsymbol{p}^2}{E_{\boldsymbol{p}}^3}\left(2 - \theta_{\boldsymbol{p}}^{(\mathrm{n})} - \theta_{\boldsymbol{p}}^{(\mathrm{p})}\right)$$

$$\to -4\pi M^2\left\{2\left(\frac{M^*}{M}\right)^2 \ln\left(\frac{M}{M^*}\right) + \left(1 - \frac{M^*}{M}\right)\left(1 - 3\frac{M^*}{M}\right)\right\} - \frac{16\pi}{15}k_{\mathrm{F}}^2 v_{\mathrm{F}}^3$$

$$< 0$$

No sea approx.

Quenchig Factor Q

$$Q = \frac{1}{\left\{1 + 2\kappa'(\bar{N})g_a/(2\pi)^3\right\}^2}\left(1 - \frac{2}{3}v_{\mathrm{F}}^2\right)$$

$$\approx \quad 1 \qquad (M^* = 0.7306M, k_{\mathrm{F}} = 1.3\mathrm{fm}^{-1})$$

No quenching

2 Dirac Sea Effects on Other Quantities

RPA in the Full Space
with Renormalization

How is the Effect of the Renormalization important for Observables ?

It depends on Observables.

1. Landau Parameters in the $\sigma - \omega$ Model

Kurasawa and Suzuki, Phys. Lett.**B474** (2000)262

No-Sea

	No $\bar{\mathrm{N}}$	Pauli Blocking	Renormalized
$F_1 =$	$-v_F^2 F_v\,,$	$-\dfrac{v_F^2 F_v}{1+\frac{1}{3}v_F^2 F_v}\,,$	$-\dfrac{v_F^2 F_v}{1+\frac{1}{3}v_F^2 F_v}$

$$F_v = N_F\left(\frac{g_v}{m_v}\right)^2, \quad v_F = p_F/E_F \quad \text{Fermi Velocity}$$

$$N_F = \frac{2p_F E_F}{\pi^2}, \quad E_F = \{p_F^2 + M^{*2}\}^{1/2} \quad \text{Density of States}$$

2. E2 GQR

$$E_{\mathrm{Q}} = \left(\frac{6p_{\mathrm{F}}^2}{5\epsilon_{\mathrm{F}}^2 \langle r^2 \rangle} \frac{1}{1 + \frac{1}{3}F_1} \right)^{1/2}$$

Nishizaki, Kurasawa and Suzuki, Nucl. Phys. A462 (87) 687

3. Magnetic Moment

$$\begin{aligned} \boldsymbol{j} &= \frac{\boldsymbol{p}}{M^*} & \text{(RMF)} \\ &\rightarrow \frac{\boldsymbol{p}}{M^*}\left(1 + \frac{1}{3}F_1\right) = \frac{\boldsymbol{p}}{M} & \text{(RRPA)} \end{aligned}$$

H. Kurasawa and T. Suzuki, Phys. Lett. 165B (85) 234

Bentz, Arima, Hyuga, Shimizu and Yazaki, N. P. A436 (85) 593.

4. Dirac Sea Effects on 0^+

Nishizaki, Kurasawa and Suzuki, Nucl. Phys. **A462** (1987) 687

$$0^+ \qquad \omega_0 = 79/A^{1/3} \quad \mathrm{MeV}$$

$$\omega_0 = \frac{1}{\epsilon_\mathrm{F}} \left\{ \frac{3p_\mathrm{F}^2}{\langle r^2 \rangle} \left(\frac{1+F_0}{1+\frac{1}{3}F_1} \right) \right\}^{1/2}$$

$$K = \frac{3p_\mathrm{F}^2}{\epsilon_\mathrm{F}} \left(\frac{1+F_0}{1+\frac{1}{3}F_1} \right), \qquad F_0 > -1$$

$F_0, \quad F_1 \quad$: **Landau Parameters**

No-Sea

No $\bar{\mathrm{N}}$ — Pauli Blocking — Renormalized

$$F_0 = F_{\mathrm{v}} - (1 - v_{\mathrm{F}}^2) F_{\mathrm{s}}, \quad F_{\mathrm{v}} - \frac{1 - v_{\mathrm{F}}^2}{1 + a_{\mathrm{s}} F_{\mathrm{s}}} F_{\mathrm{s}}, \quad F_{\mathrm{v}} - \frac{1 - v_{\mathrm{F}}^2}{1 + a_{\mathrm{s}} F_{\mathrm{s}} + a_{\mathrm{D}}} F_{\mathrm{s}}$$

$F_0 < -1$ $\qquad$ $a_{\mathrm{s}} F_{\mathrm{s}} < a_{\mathrm{D}}$

$$F_{\mathrm{s}} = N_{\mathrm{F}} \left(\frac{g_{\mathrm{s}}}{m_{\mathrm{s}}}\right)^2, \quad F_{\mathrm{v}} = N_{\mathrm{F}} \left(\frac{g_{\mathrm{v}}}{m_{\mathrm{v}}}\right)^2, \quad v_{\mathrm{F}} = p_{\mathrm{F}} / E_{\mathrm{F}} \quad \text{Fermi Velocity}$$

$$N_{\mathrm{F}} = \frac{2 p_{\mathrm{F}} E_{\mathrm{F}}}{\pi^2}, \quad E_{\mathrm{F}} = \{p_{\mathrm{F}}^2 + M^{*2}\}^{1/2} \quad \text{Density of States}$$

$$a_{\mathrm{s}} = \frac{3}{2}\left(1 - \frac{2}{3} v_{\mathrm{F}}^2 + \frac{1 - v_{\mathrm{F}}^2}{2 v_{\mathrm{F}}} \ln \frac{1 - v_{\mathrm{F}}}{1 + v_{\mathrm{F}}}\right)$$

$$a_{\mathrm{D}} = \frac{1}{\pi^2} \left(\frac{g_{\mathrm{s}}}{m_{\mathrm{s}}}\right)^2 \left[3 M^{*2} \ln \frac{M}{M^*} + M^2 a_2 + 3M(M^* - M) + \frac{9}{2}(M^* - M)^2\right],$$

$$a_2 = \frac{m_{\mathrm{s}}^2}{4M^2} + \frac{3}{2} \int_0^1 dx \ln \left\{1 - \frac{m_{\mathrm{s}}^2}{M^2} x(1 - x)\right\},$$

3. Conclusions

The renormalization is necessary .

The No-Sea Approximation is not justified sometimes.

Future Problems for Rel. Models :

How do we renormalize other divergences ?

Which is a real phenomenological model, Rel. or Non-Rel. Model ?