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Effects of the Dirac Sea on Giant Resonance States in Relativistic Nuclear Models

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Purpose

1. Sum Rules require Anti-Nucleons.
2. Effects of the Renormalization should be examined.

Example : Gamow-Teller Sum Rules

K. Ikeda, S. Fujii and J.I. Fujita, Phys. Lett. 3, 271 (1963)

Gamow-Teller Sum Rule (Ikeda Sum Rule)

In Non-relativistic models

$$Q_{\pm} = \sum_{i}^{A} \tau_{\pm i} \sigma_{yi} \qquad \tau_{\pm} = (\tau_x \pm i\tau_y)/\sqrt{2} \quad , \qquad \tau_0 = \tau_z .$$

Sum Rule for β_- and β_+ transitions

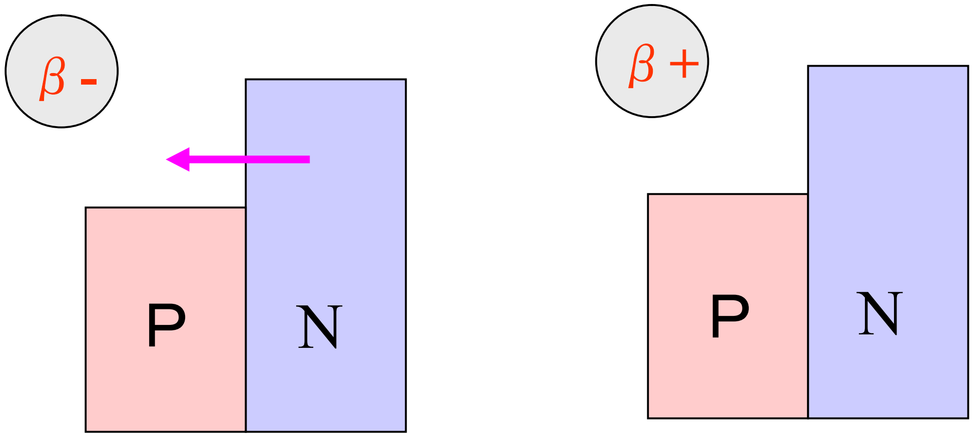
$$\langle \, | Q_+ Q_- | \, \rangle - \langle \, | Q_- Q_+ | \, \rangle = 2(N - Z),$$

because

$$[\, \tau_+ \sigma_y \, , \, \tau_- \sigma_y \,] = 2\tau_z .$$

GT Transitions in the Relativistic Mean Field

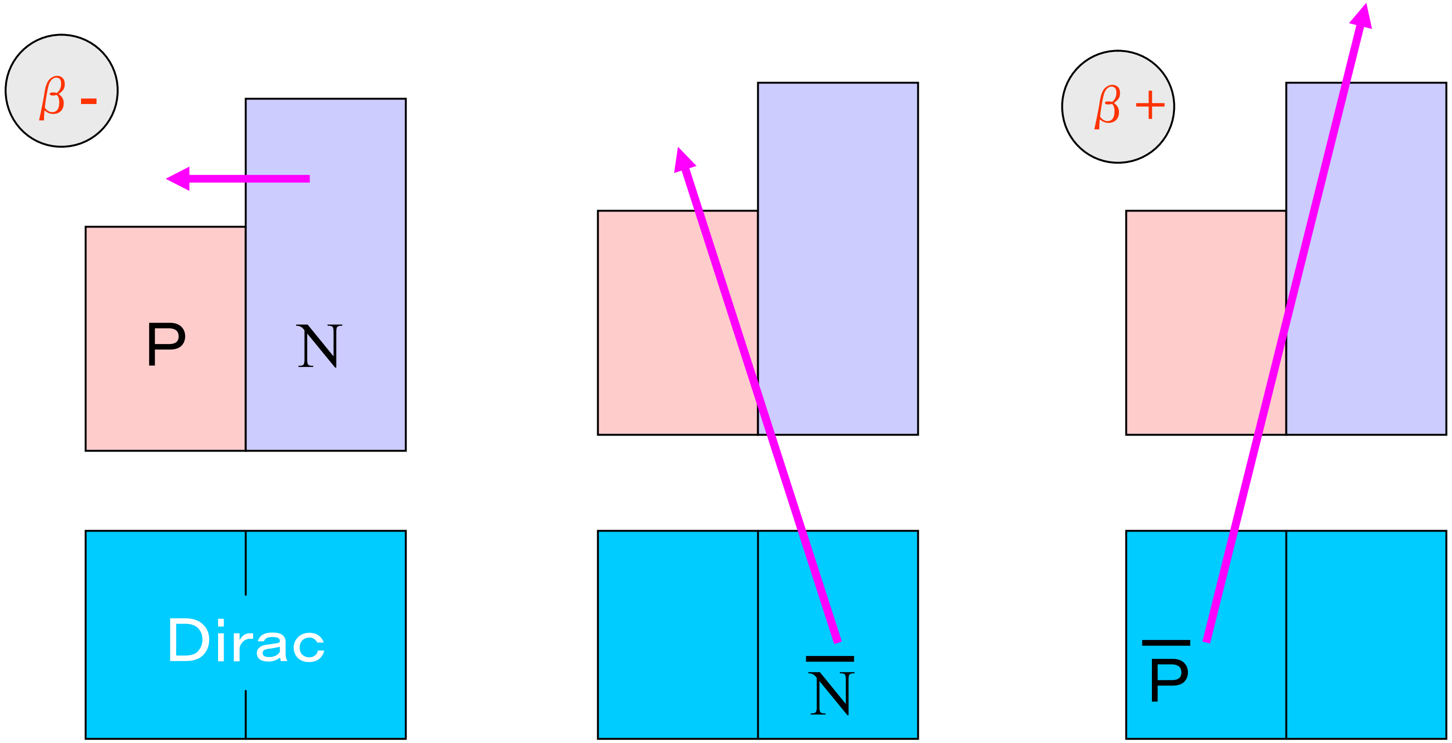
$$A - 0 \neq 2(N-Z)$$



H. Kurasawa, T. Suzuki and N.V. Giai, Phys. Rev. Lett. 91, 062501(2003);
Phys. Rev. C68, 064311 (2004)

GT Transitions in the Relativistic Mean Field

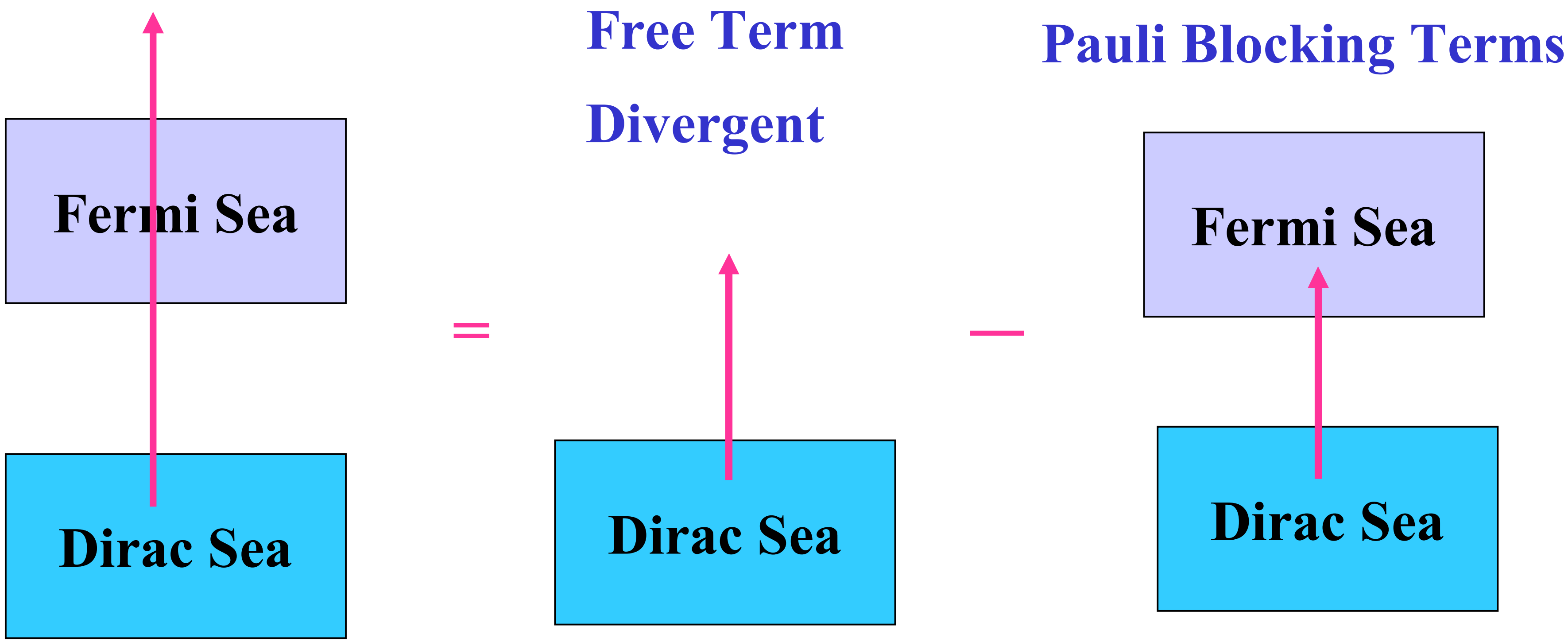
$$A \quad + \quad (B + \infty) \quad - \quad (C + \infty) \;=\; 2(N\text{-}Z)$$



S.A. Chin, Ann. of Phys. 108,301(1977)
H. Kurasawa and T. Suzuki, Nucl. Phys. A445,685 (1985)

RPA without Renormalization

RPA I (No Free Term Approx.)



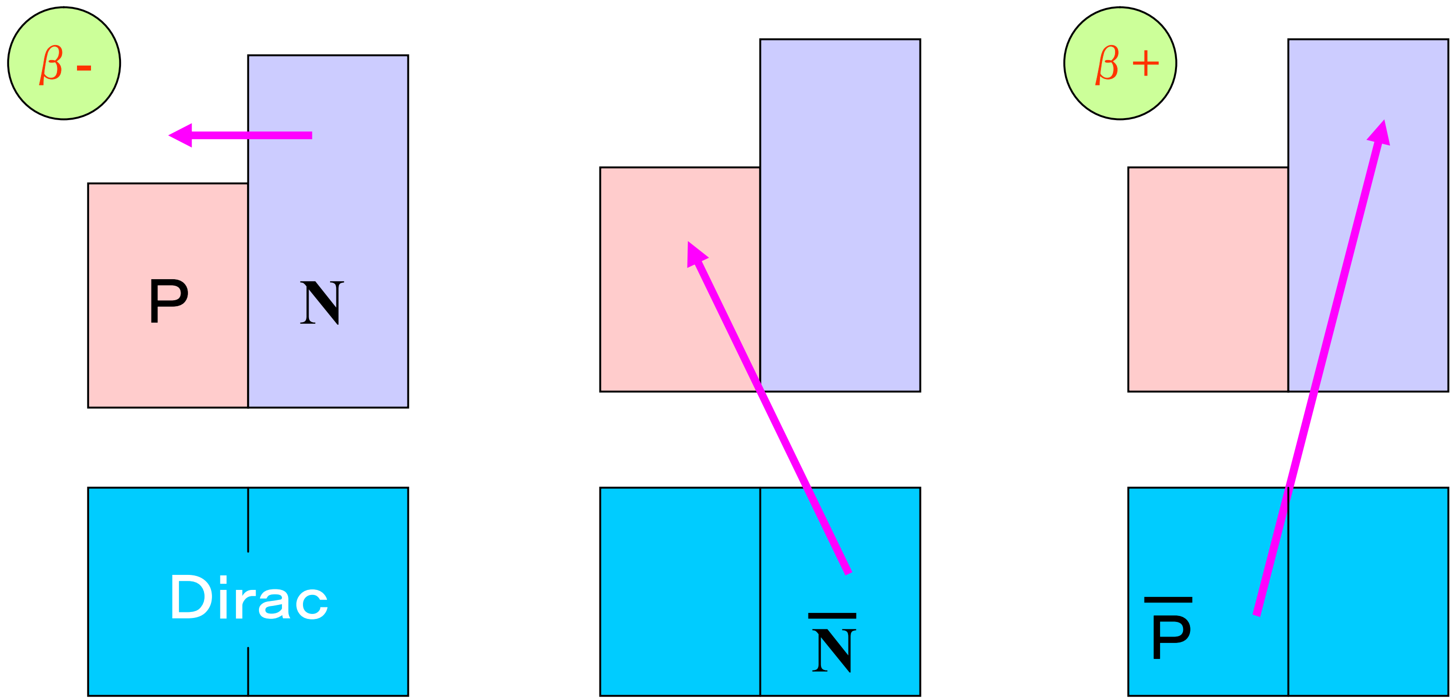
Current Conservation is satisfied.

H. Kurasawa and T. Suzuki, Phys. Rev. C69, 014306 (2004)

GT Transition in RPA I (No Free Term Approx.)

A' + (- B') - (- C') = 2(N-Z)

Pauli Blocking Terms



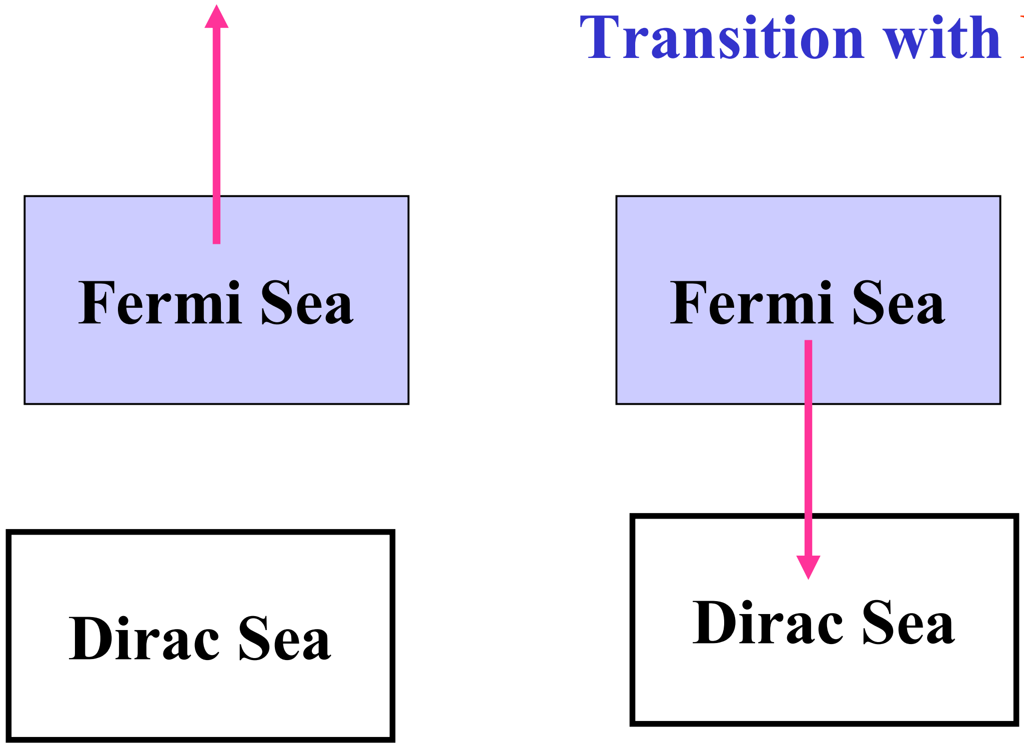
Ikeda sum rule is satisfied.

J.F. Dawson and R.J. Furnstahl, Phys. Rev. C42, 2009 (1990) ; Z. Ma et al., Nucl. Phys. A685, 173 (2001)

RPA without Renormalization

RPA II (No Sea Approx.)

Transition with Negative Energy



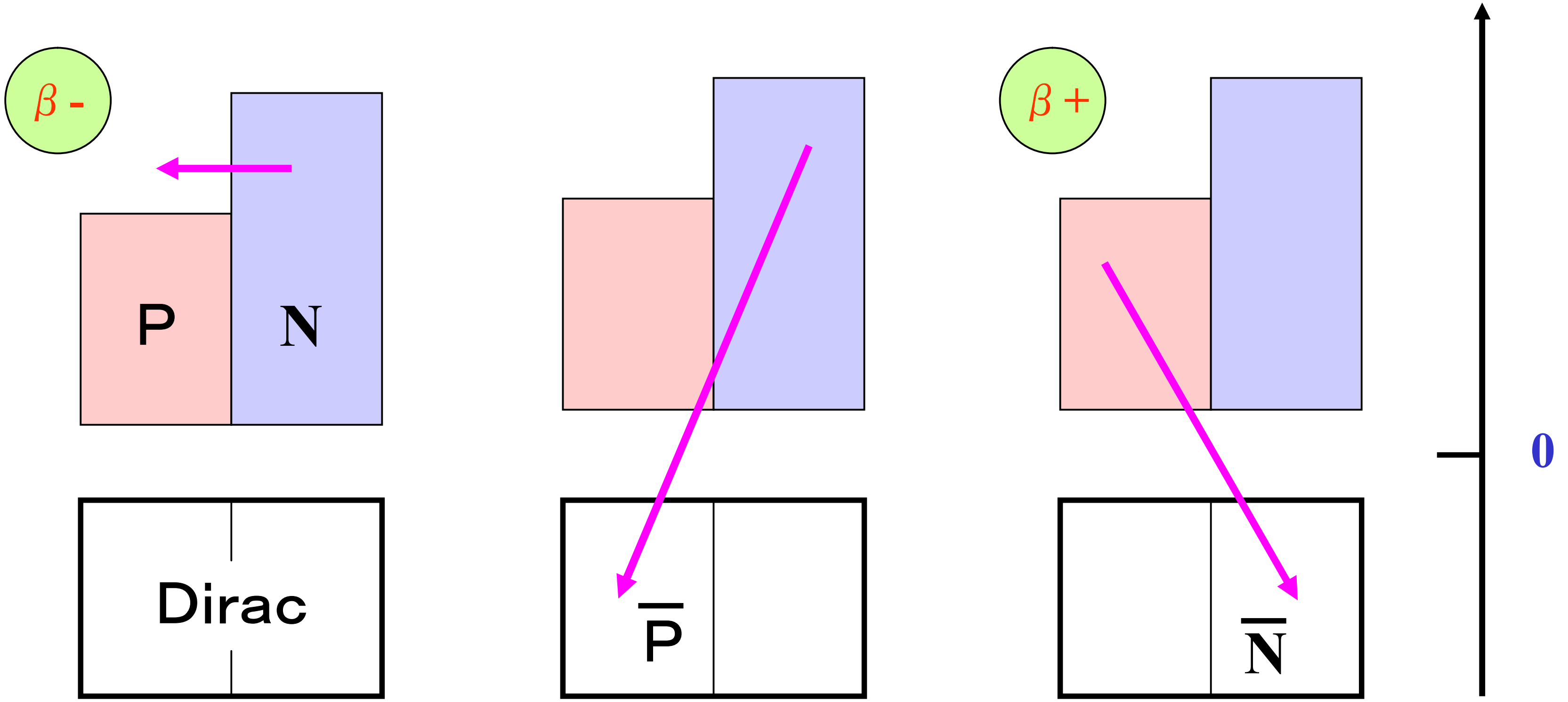
Current Conservation is satisfied.

H. Kurasawa and T. Suzuki, Phys. Rev. C69, 014306 (2004)

GT Transition in RPA II (No Sea Approx.)

A' + (C') - (B') = 2(N-Z)

Ex. E > 0 | Ex. E < 0 | Ex. E < 0 | E



D.J. Thouless, Nucl. Phys. 22, 78 (1961)

Energy-Weighted Sum

RPA Theorem by Thouless
in Non-Relativistic and No Charge-Exchange Excitations

Energy-Weighted Sum of the Strengths in RPA

$$= < \; | [F , [H , F]] | \; >$$

We can also prove the same theorem for the sum of the β - and β + transition strengths in Relativistic RPA I and II .

H. Kurasawa and T. Suzuki, Phys. Rev. C69, 014306 (2004)

Summary of RPA I and II

If we include a part of antinucleon degrees of freedom,

1. Ikeda-Fujii-Fujita Sum Rule is satisfied.

2. RPA theorem also holds for the sum of β- and β+.

H. Kurasawa and T. Suzuki, Phys. Rev. C69, 014306 (2004)

Moreover,

3. In the giant resonance region, RPA seems to work.

But there is a serious problem of RPA I and II.

The sum rules are satisfied, owing to the negative energy-weighted strengths from the anti-nucleons.

Outside of the Giant Resonance Region

	Strength	Energy	Energy-Weighted Strength
RPA I	negative	positive	negative
RPA II	positive	negative	negative

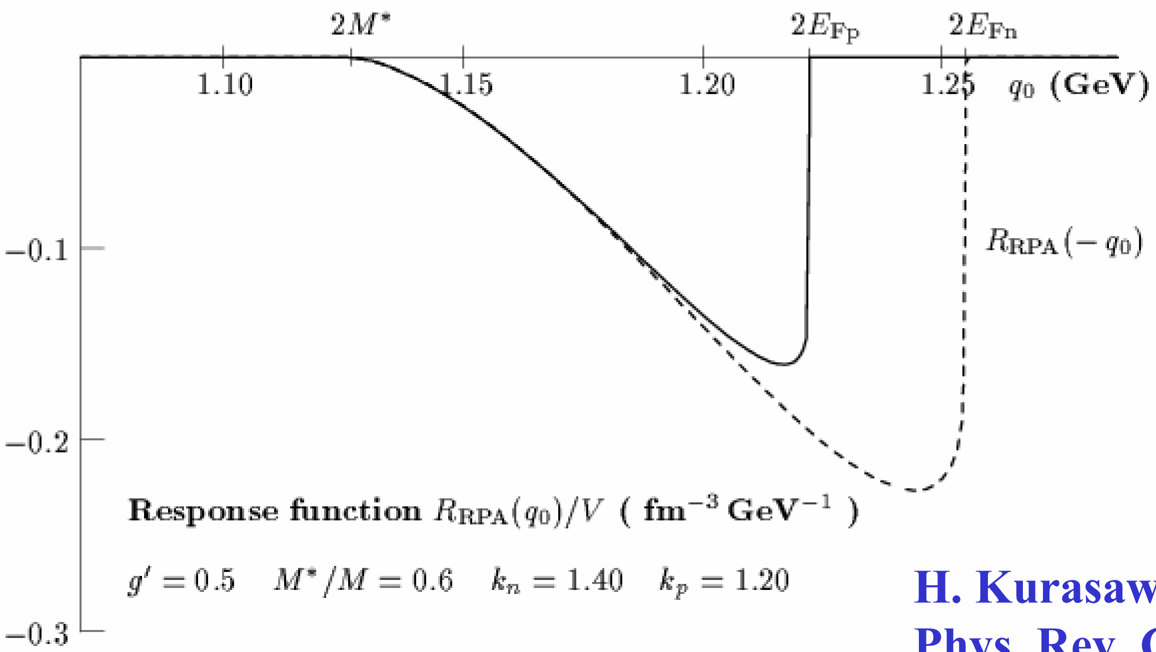
Table 1 $S_{\text{total}} = S_{\text{GT}} + S^{(-)}_{\text{NFA}} - S^{(+)}_{\text{NFA}}$, V : nuclear volume

g'	ω_0 (MeV)	S_{GT}/V	$S^{(-)}_{\text{NFA}}/V$	$S^{(+)}_{\text{NFA}}/V$	S_{total}/V
0.00	0.0	0.06072	−0.00733	−0.01524	0.06863
0.50	11.9440	0.06116	−0.00750	−0.01497	0.06863
1.00	23.9751	0.06161	−0.00767	−0.01470	0.06863
2.00	48.2999	0.06250	−0.00804	−0.01417	0.06863
3.00	72.9773	0.06341	−0.00844	−0.01366	0.06863
4.00	98.0117	0.06432	−0.00887	−0.01318	0.06863
5.00	123.4095	0.06526	−0.00934	−0.01271	0.06863

Example :

Non Energy-Weighted Strength in Nucl. Matter

Sum Rule : S(total)/V=0.06863
S(GT) / S(total) = 88%



H. Kurasawa and T. Suzuki,
Phys. Rev. C69, 014306 (2004)

Fig. 1 $E_{\text{Fn(p)}} = \sqrt{M^{*2} + k_{n(p)}}$ $k_{n(p)}$: Fermi momentum of neutron (proton)

We have unphysical results in RPA I and II,

as a price of Neglecting the Divergence.

We need the renormalization of the Divergence.

How is the Effect of the Renormalization important for Observables ?

(For GT states, we do not know the answer yet.)

It depends on Observables in Previous Results.

Landau Parameters in the $\sigma - \omega$ Model

Kurasawa and Suzuki, Phys. Lett.**B474** (2000) 262

	No $\bar{\mathrm{N}}$	**RPA I, II**	Renormalized
$F_1 =$	$-v_{\mathrm{F}}^2 F_{\mathrm{v}}\,,$	$-\dfrac{v_{\mathrm{F}}^2 F_{\mathrm{v}}}{1+\frac{1}{3}v_{\mathrm{F}}^2 F_{\mathrm{v}}}\,,$	$-\dfrac{v_{\mathrm{F}}^2 F_{\mathrm{v}}}{1+\frac{1}{3}v_{\mathrm{F}}^2 F_{\mathrm{v}}}$
		Correct	**Correct**

$$F_{\mathrm{v}} = N_{\mathrm{F}}\left(\frac{g_{\mathrm{v}}}{m_{\mathrm{v}}}\right)^2, \quad v_{\mathrm{F}} = p_{\mathrm{F}}/E_{\mathrm{F}} \qquad \text{Fermi Velocity}$$

$$N_{\mathrm{F}} = \frac{2p_{\mathrm{F}}E_{\mathrm{F}}}{\pi^2}, \qquad E_{\mathrm{F}} = \{p_{\mathrm{F}}^2 + M^{*2}\}^{1/2} \qquad \text{Density of States}$$

F_1 - Dependent Observables

1. E1 : Center of Mass Motion

$$
\begin{aligned}
E(v) &\approx E(0) + \frac{1}{2} v^2 A \sqrt{M^{*2} + p_{\mathrm{F}}^2} \quad (\text{ RMF }) \\
&\rightarrow E(0) + \frac{1}{2} v^2 A \sqrt{M^{*2} + p_{\mathrm{F}}^2} \Big/ \left(1 + \frac{1}{3} F_1\right) \\
&= E(0) + \frac{1}{2} v^2 A \epsilon_{\mathrm{F}} \quad (\text{ RRPA })
\end{aligned}
$$

ϵ_{F} Fermi Energy

Nishizaki, Kurasawa and Suzuki, Nucl. Phys. A462 (87) 687

J. F. Dawson and R. J. Furnstahl, Phys. Rev. C42 (90) 2009

2. E2 GQR

$$E_{\mathrm{Q}} = \left(\frac{6p_{\mathrm{F}}^2}{5\epsilon_{\mathrm{F}}^2 \langle r^2 \rangle} \frac{1}{1 + \frac{1}{3}F_1} \right)^{1/2}$$

Nishizaki, Kurasawa and Suzuki, Nucl. Phys. A462 (87) 687

3. Magnetic Moment

$$\begin{aligned} \boldsymbol{j} &= \frac{\boldsymbol{p}}{M^*} && (\text{ RMF }) \\ &\rightarrow \frac{\boldsymbol{p}}{M^*}\left(1 + \frac{1}{3}F_1\right) = \frac{\boldsymbol{p}}{M} && (\text{ RRPA }) \end{aligned}$$

H. Kurasawa and T. Suzuki, Phys. Lett. 165B (85) 234

Bentz, Arima, Hyuga, Shimizu and Yazaki, N. P. A436 (85) 593.

No $\bar{\mathrm{N}}$ | **RPA I, II** | Renormalized

$$F_0 = F_\mathrm{v} - (1 - v_\mathrm{F}^2)F_\mathrm{s}, \quad F_\mathrm{v} - \frac{1 - v_\mathrm{F}^2}{1 + a_\mathrm{s}F_\mathrm{s}}F_\mathrm{s}, \quad F_\mathrm{v} - \frac{1 - v_\mathrm{F}^2}{1 + a_\mathrm{s}F_\mathrm{s} + a_\mathrm{D}}F_\mathrm{s}$$

$F_0 < -1$ $\qquad$ $a_\mathrm{s}F_\mathrm{s} < a_\mathrm{D}$

$$F_\mathrm{s} = N_\mathrm{F}(\frac{g_\mathrm{s}}{m_\mathrm{s}})^2, \quad F_\mathrm{v} = N_\mathrm{F}(\frac{g_\mathrm{v}}{m_\mathrm{v}})^2, \quad v_\mathrm{F} = p_\mathrm{F}/E_\mathrm{F} \quad \text{Fermi Velocity}$$

$$N_\mathrm{F} = \frac{2p_\mathrm{F}E_\mathrm{F}}{\pi^2}, \quad E_\mathrm{F} = \{p_\mathrm{F}^2 + M^{*2}\}^{1/2} \quad \text{Density of States}$$

$$a_\mathrm{s} = \frac{3}{2}(1 - \frac{2}{3}v_\mathrm{F}^2 + \frac{1 - v_\mathrm{F}^2}{2v_\mathrm{F}}\ln\frac{1 - v_\mathrm{F}}{1 + v_\mathrm{F}})$$

$$a_\mathrm{D} = \frac{1}{\pi^2}\left(\frac{g_\mathrm{s}}{m_\mathrm{s}}\right)^2\left[3M^{*2}\ln\frac{M}{M^*} + M^2 a_2 + 3M(M^* - M) + \frac{9}{2}(M^* - M)^2\right],$$

$$a_2 = \frac{m_\mathrm{s}^2}{4M^2} + \frac{3}{2}\int_0^1 dx\ln\left\{1 - \frac{m_\mathrm{s}^2}{M^2}x(1-x)\right\},$$

Giant Monopole States in Relativistic Models

Nishizaki, Kurasawa and Suzuki, Nucl. Phys. **A462** (1987) 687

$$\omega_0 = \frac{1}{\epsilon_{\rm F}}\left\{\frac{3p_{\rm F}^2}{\langle r^2\rangle}\left(\frac{1+F_0}{1+\frac{1}{3}F_1}\right)\right\}^{1/2}$$

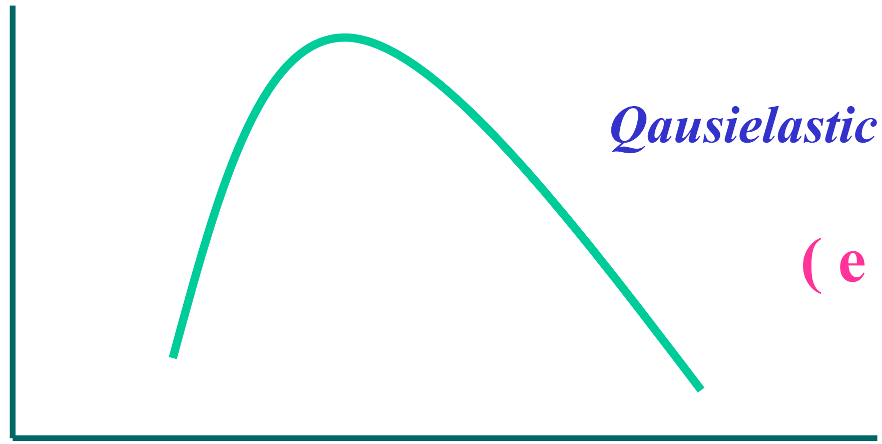
$F_0, \quad F_1 \quad :$ **Landau Parameters**

$$F_0 > -1$$

$$K = \frac{3p_{\rm F}^2}{\epsilon_{\rm F}}\left(\frac{1+F_0}{1+\frac{1}{3}F_1}\right)$$

$S_L(q)$

$q = (\mathbf{q}, q_0 = \omega)$



0

ω

Qausielastic Electron Scattering

(e, e')

Coulomb Sum Value

$$C(|\boldsymbol{q}|) = \frac{1}{Z}\int_0^{|\boldsymbol{q}|} dq_0 S_{\mathrm{L}}(q)$$

$$S_{\mathrm{L}}(q) = \sum_n |\langle n|J_0(q)|0\rangle|^2 \delta(E_n - q_0)$$

$$\sim \mathrm{Im}\Pi(J_0, J_0)$$

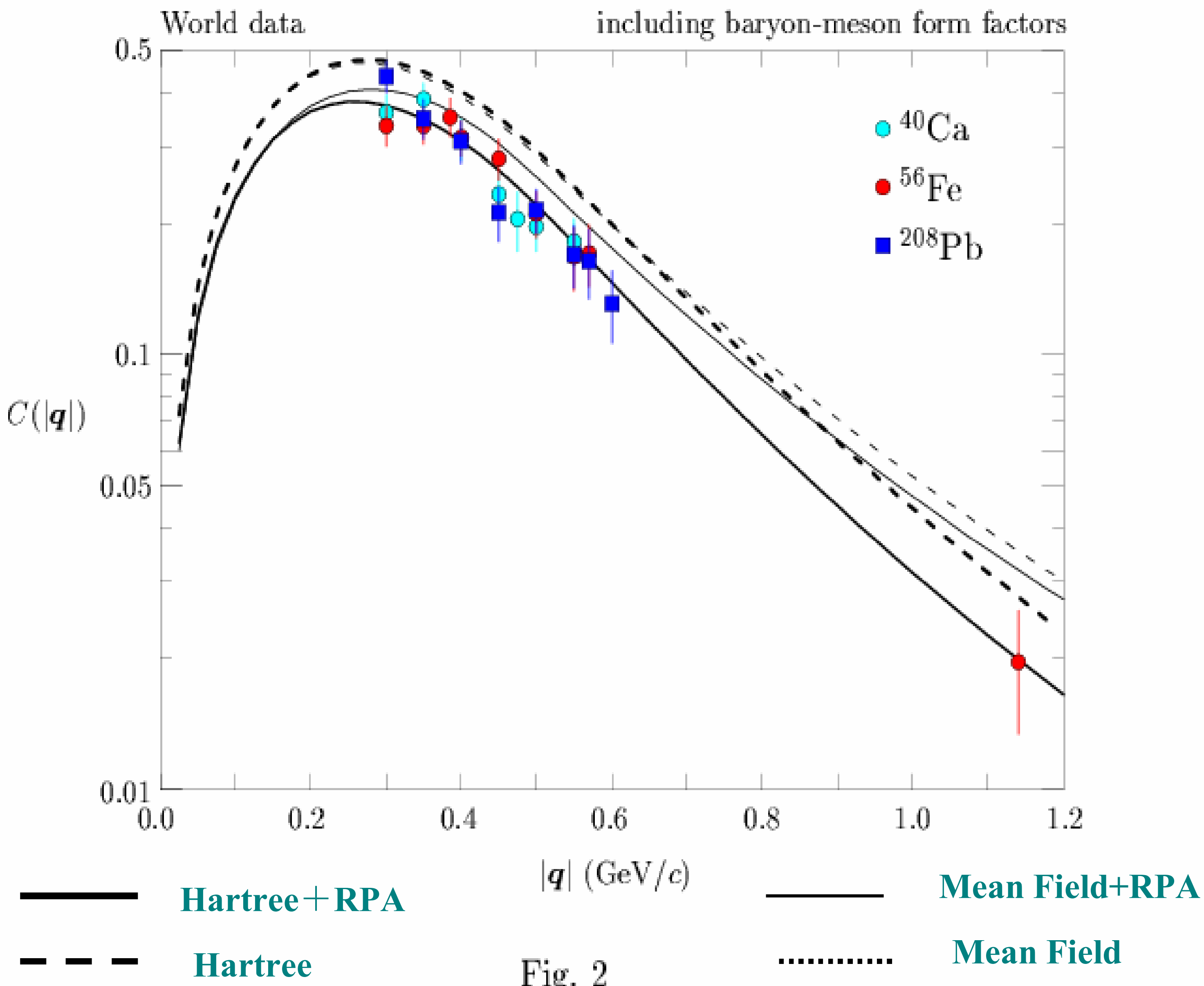


Fig. 2

See also H. Kurasawa and T. Suzuki, N. P. A490 (1988) 571 ; P. T. P. 86 (1991) 773.
C. J. Horowitz and J. Piekarewicz, P. R. L. 62 (1989) 391.

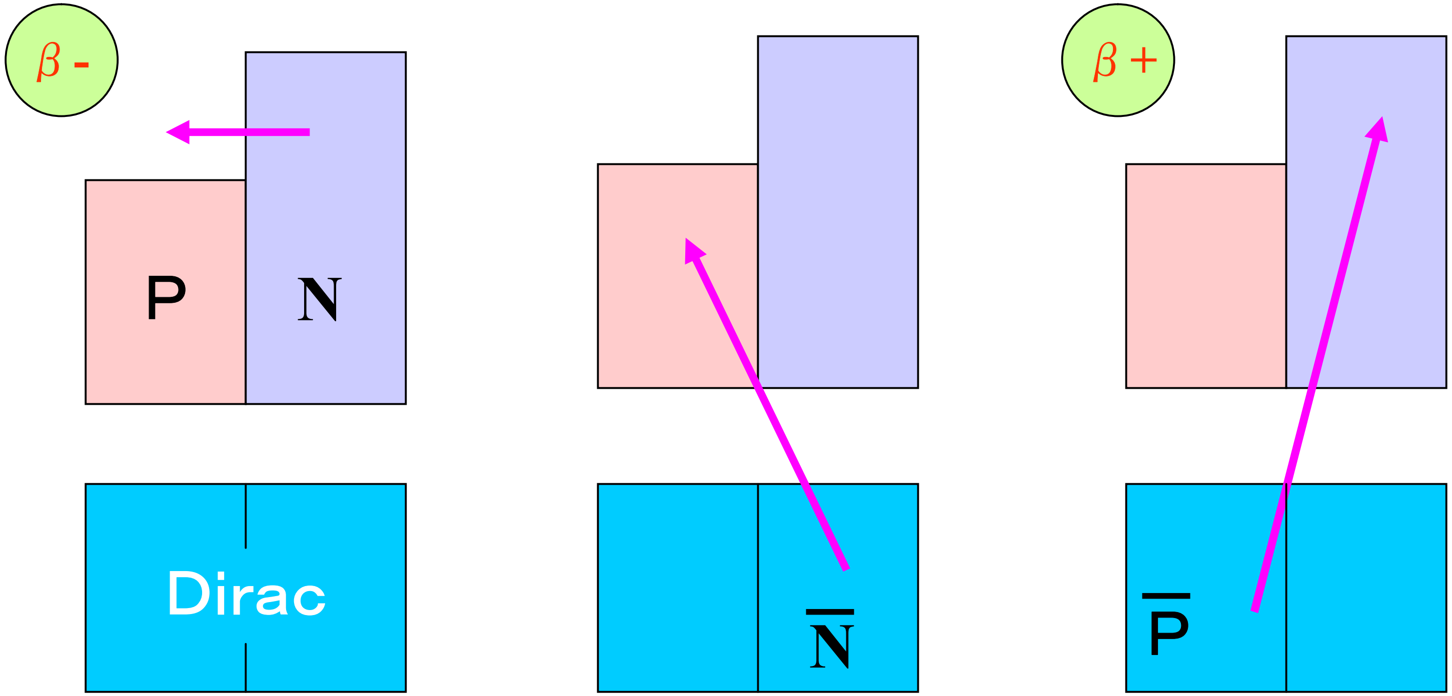
GT Transition

How is the renormalizatio important for GT ?

I do not know the answer, But it is important question.

$$A' + (-B') - (-C') = 2(N-Z)$$

Pauli Blocking Terms



H. Kurasawa, T. Suzuki and N.V. Giai, Phys. Rev. Lett. 91, 062501(2003)

The Strength of Giant GT Resonance State

$$|\langle \mathrm{GT} \,|\, F_- \,|\, \rangle|^2 \approx 2\left(1 - \frac{2}{3}v_\mathrm{F}^2\right)(N - Z)$$

A’

v_F : Fermi velocity

The classical sum value is quenched by 12%

for $v_\mathrm{F} = 0.43 \quad (M^* = 0.6M, \quad k_\mathrm{F} = 1.36\mathrm{fm}^{-1})$.

(6% in finite nuclei, H. Kurasawa et al., Phys. Rev. 68, 064311 (2003);
Z. Ma et al., in press in EPJ A)

This is small, but important.

C. Gaarde, Nucl. Phys. A396 (1983) 127c

Old ～ 50 %

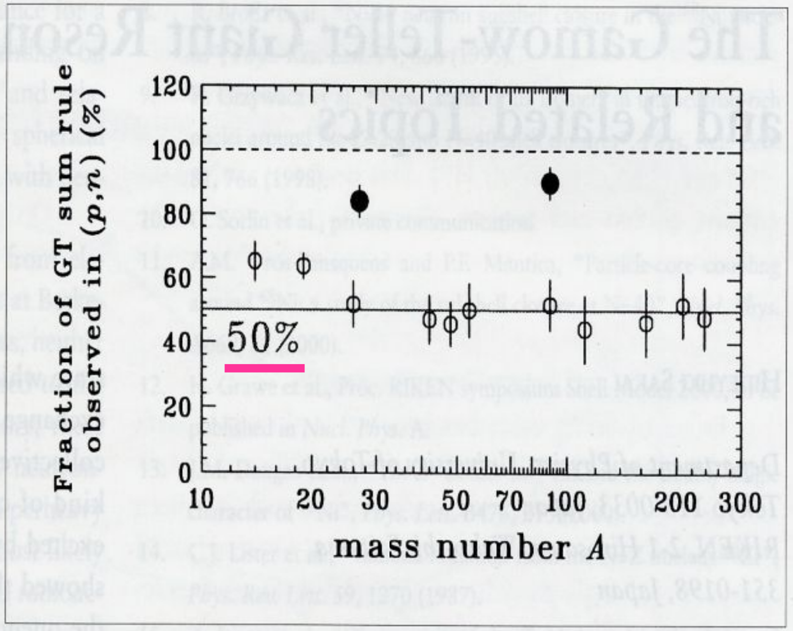


Figure 1. *Fraction of the GT sum-rule value taken from Gaarde [9] (open circle). Solid circles are new measurements taken from [18, 24].*

T. Wakasa et al., Phys. Rev. C55 (1997) 2909

Phys. Lett. B426 (1998) 257

New 84 ～ 90 %

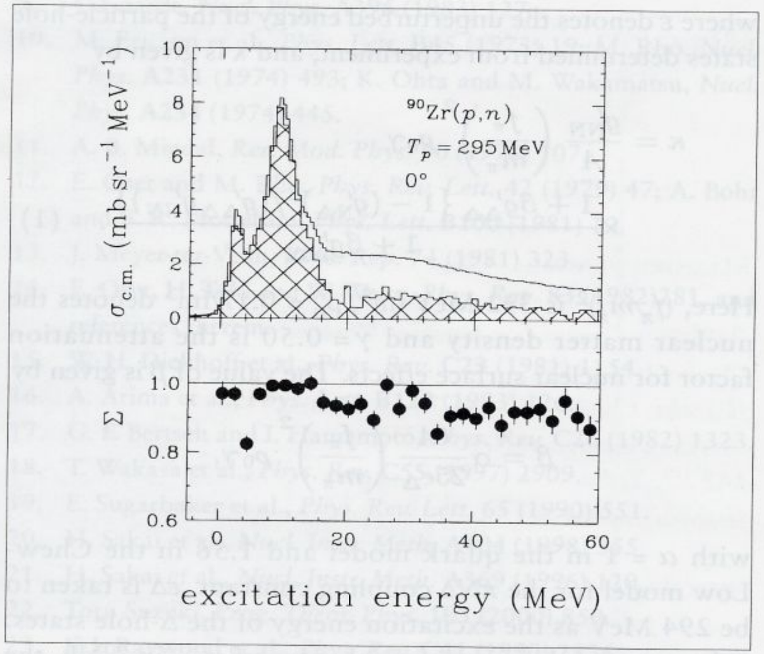
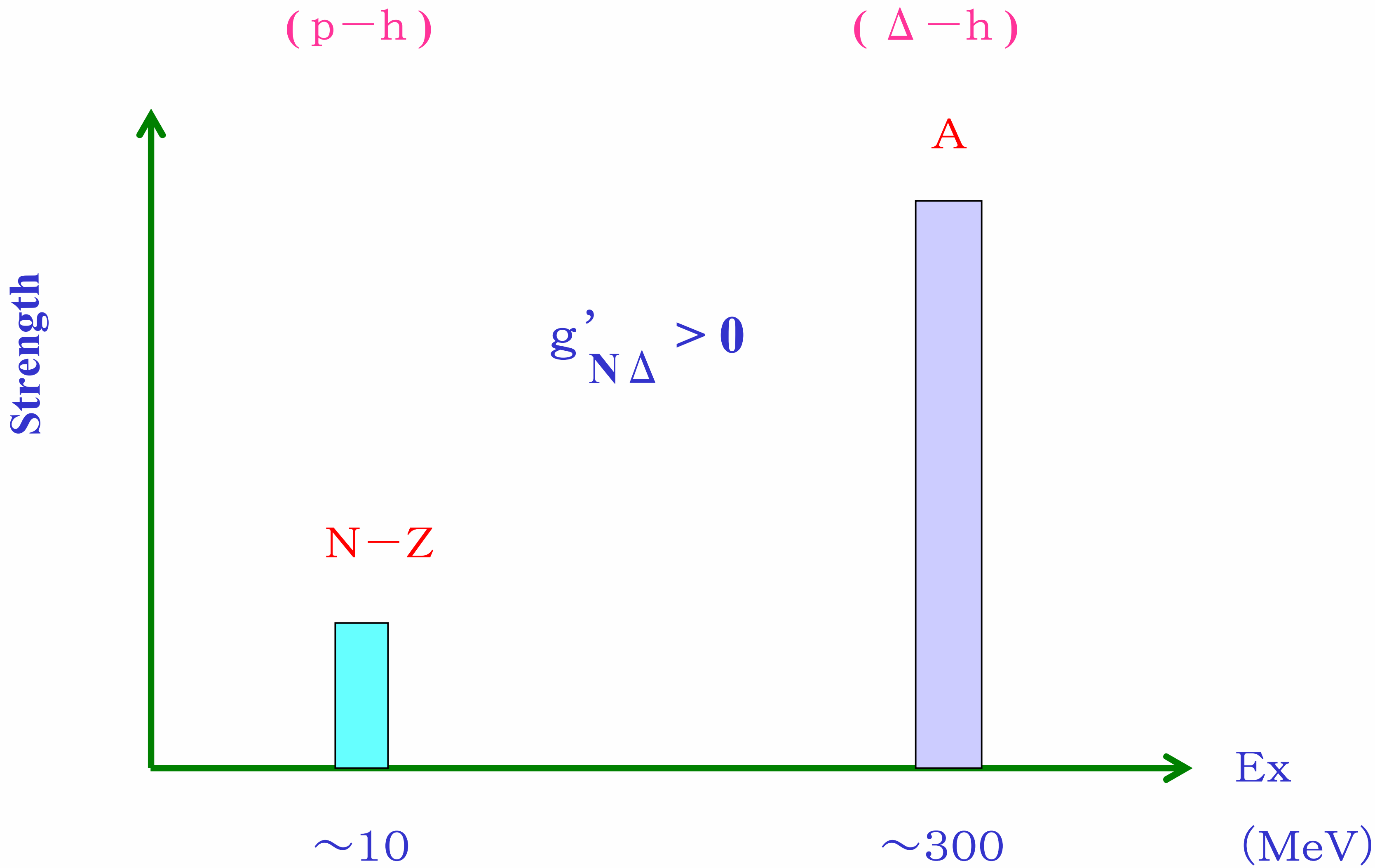


Figure 2. *The energy spectrum (upper) and the total spin-transfer Σ (lower) for the $^{90}Zr(p,n)$ reaction at T_p = 295 MeV and θ_{lab} = 0°.*

Previous Understanding :

Coupling of the Particle-Hole States with Δ - Hole States



	Quenching	g'(N Δ)	Pion condensation critical density/ ρ_0
<1997	40%	0.75 (quark model)	> 8
>1997	10%	0.21 (quark model)	< 2
	4%	0.12 (quark model)	～ 1

(6% due to the anti-nucleons)

T. Suzuki, H. Sakai and T. Tatsumi, Proc. of the Int. Workshop on Nucl. Res. and Medium Effects (Universal Academy Press, Tokyo 1999) p77

Conclusions

1. Various laws are satisfied, if a part of anti-nucleon degrees of freedom are taken into account.

2. As a price of neglecting the divergence, however, we obtain unphysical results also.

3. We should estimate effects of the renormalization on Observables.